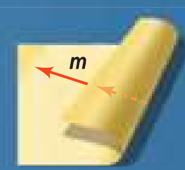
# 3-6 Constructing Parallel and Perpendicular Lines

**Objective** To construct parallel and perpendicular lines

Check out the angles formed by the folded lines.

**SOLVE IT!** Getting Ready!

Draw a line m on a sheet of paper. Fold your paper so that line m falls on itself. Label your fold line n. Fold your paper again so that n falls on itself. Label your new fold line p. How are m and p related? How do you know?




In the Solve It, you used paper-folding to construct lines.

**Dynamic Activity** Constructing Parallel and Perpendicular Lines

**Essential Understanding** You can also use a straightedge and a compass to construct parallel and perpendicular lines.

In Lesson 3-5, you learned that through a point not on a line, there is a unique line parallel to the given line. Problem 1 shows the construction of this line.

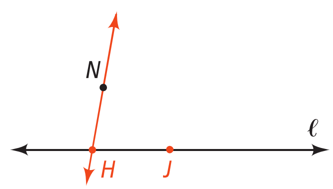
## Problem 1 Constructing Parallel Lines

**Construct the line parallel to a given line and through a given point that is not on the line.**

**Given:** line $\ell$ and point $N$ not on $\ell$

**Construct:** line $m$ through $N$ with $m \parallel \ell$


**Step 1** Label two points $H$ and $J$ on $\ell$. Draw $\overleftrightarrow{HN}$.

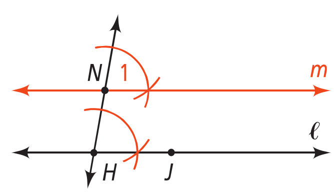



**Think**

**What type of angles are $\angle 1$ and $\angle NHJ$?**
They are corresponding angles.

**Step 2** At $N$, construct $\angle 1$ congruent to $\angle NHJ$. Label the new line $m$.

$m \parallel \ell$




**Got It?** 1. **Reasoning** Why must lines $\ell$ and $m$ be parallel?

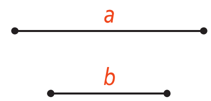
## Problem 2 Constructing a Special Quadrilateral

**Plan**

**How do you know which constructions to use?**
Try sketching the final figure. This can help you visualize the construction steps you will need.

**Construct a quadrilateral with one pair of parallel sides of lengths $a$ and $b$.**

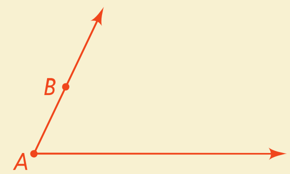
**Given:** segments of lengths $a$ and $b$



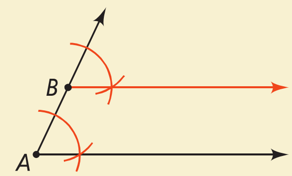

**Construct:** quadrilateral $ABYZ$ with
$AZ = a$, $BY = b$, and $\overleftrightarrow{AZ} \parallel \overleftrightarrow{BY}$

**Think** / **Write**

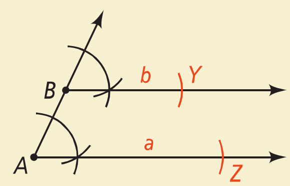
You need a pair of parallel sides, so construct parallel lines as you did in Problem 1. Start by drawing a ray with endpoint $A$. Then draw $\overrightarrow{AB}$ such that point $B$ is not on the first ray.



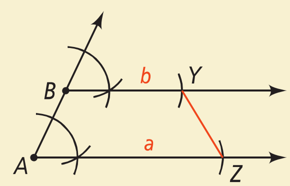

Construct congruent corresponding angles to finish your parallel lines.




Now you need sides of lengths $a$ and $b$. In Lesson 1-6, you learned how to construct congruent segments. Construct $Y$ and $Z$ so that $BY = b$ and $AZ = a$.




Draw $\overline{YZ}$.




$ABYZ$ is a quadrilateral with parallel sides of lengths $a$ and $b$.

**Got It?** **2. a.** Draw a segment. Label its length $m$. Construct quadrilateral $ABCD$ with $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, so that $AB = m$ and $CD = 2m$.

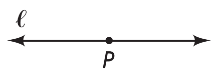
**b. Reasoning** Suppose you and a friend both use the steps in Problem 2 to construct $ABYZ$ independently. Will your quadrilaterals necessarily have the same angle measures and side lengths? Explain.

## Problem 3 Perpendicular at a Point on a Line

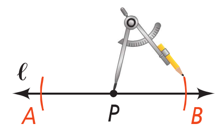
**Construct the perpendicular to a given line at a given point on the line.**

**Given:** point $P$ on line $\ell$

**Construct:** $\overleftrightarrow{CP}$ with $\overleftrightarrow{CP} \perp \ell$



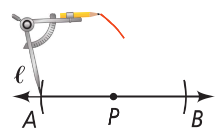

**Step 1** Construct two points on $\ell$ that are equidistant from $P$. Label the points $A$ and $B$.



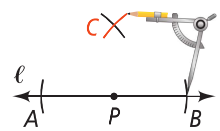

**Think**

**Why is it important to open your compass wider?**
If you don't, you won't be able to draw intersecting arcs above point *P*.

**Step 2** Open the compass wider so the opening is greater than $\frac{1}{2}AB$. With the compass tip on $A$, draw an arc above point $P$.

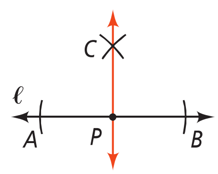



**Step 3** Without changing the compass setting, place the compass point on point $B$. Draw an arc that intersects the arc from Step 2. Label the point of intersection $C$.




**Step 4** Draw $\overleftrightarrow{CP}$.

$\overleftrightarrow{CP} \perp \ell$



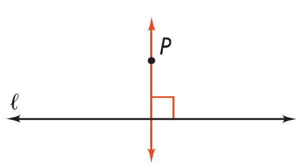

**Got It?** **3.** Use a straightedge to draw $\overleftrightarrow{EF}$. Construct $\overleftrightarrow{FG}$ so that $\overleftrightarrow{FG} \perp \overleftrightarrow{EF}$ at point $F$.

You can also construct a perpendicular line from a point to a line. This perpendicular line is unique according to the Perpendicular Postulate. You will prove in Chapter 5 that the shortest path from any point to a line is along this unique perpendicular line.

**take note**

### Postulate 3-4 Perpendicular Postulate

Through a point not on a line, there is one and only one line perpendicular to the given line.




There is exactly one line through $P$ perpendicular to $\ell$.

## Problem 4 Perpendicular From a Point to a Line

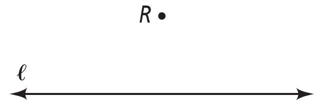
### Plan

**Can you use similar steps for this problem as in Problem 3?**
Yes. Mark two points on $\ell$ that are equidistant from the given point. Then draw two intersecting arcs.

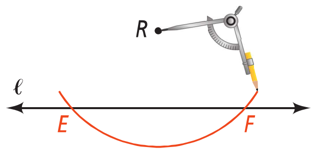
**Construct the perpendicular to a given line through a given point not on the line.**

**Given:** line $\ell$ and point $R$ not on $\ell$

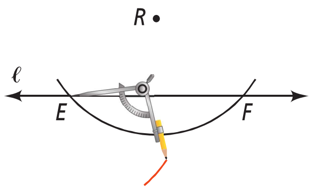
**Construct:** $\overleftrightarrow{RG}$ with $\overleftrightarrow{RG} \perp \ell$



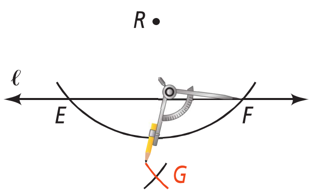

**Step 1** Open your compass to a size greater than the distance from $R$ to $\ell$. With the compass on point $R$, draw an arc that intersects $\ell$ at two points. Label the points $E$ and $F$.




**Step 2** Place the compass point on $E$ and make an arc.

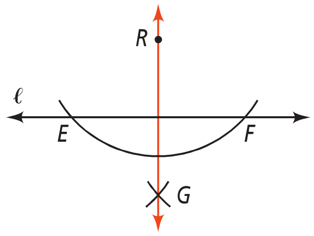



**Step 3** Keep the same compass setting. With the compass tip on $F$, draw an arc that intersects the arc from Step 2. Label the point of intersection $G$.




**Step 4** Draw $\overleftrightarrow{RG}$.

$\overleftrightarrow{RG} \perp \ell$




**Got It?** **4.** Draw $\overleftrightarrow{CX}$ and a point $Z$ not on $\overleftrightarrow{CX}$. Construct $\overleftrightarrow{ZB}$ so that $\overleftrightarrow{ZB} \perp \overleftrightarrow{CX}$.

## Lesson Check

### Do you know HOW?

**1.** Draw a line $\ell$ and a point $P$ not on the line. Construct the line through $P$ parallel to line $\ell$.

**2.** Draw $\overleftrightarrow{QR}$ and a point $S$ on the line. Construct the line perpendicular to $\overleftrightarrow{QR}$ at point $S$.

**3.** Draw a line $w$ and a point $X$ not on the line. Construct the line perpendicular to line $w$ at point $X$.

### Do you UNDERSTAND?

**4.** In Problem 3, is $\overline{AC}$ congruent to $\overline{BC}$? Explain.

**5.** Suppose you use a wider compass setting in Step 1 of Problem 4. Will you construct a different perpendicular line? Explain.

**6.** **Compare and Contrast** How are the constructions in Problems 3 and 4 similar? How are they different?

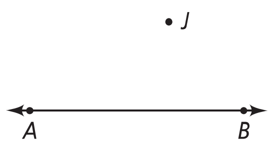
## Practice and Problem-Solving Exercises

**A Practice**

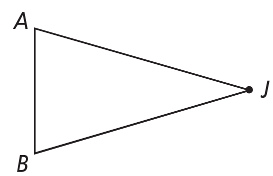
**For Exercises 7–10, draw a figure like the given one. Then construct the line through point $J$ that is parallel to $\overleftrightarrow{AB}$.**

**See Problem 1.**

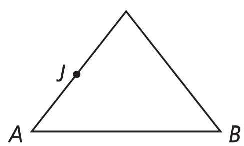
**7.**



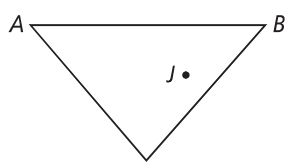

**8.**




**9.**




**10.**




**For Exercises 11–13, draw two segments. Label their lengths $a$ and $b$. Construct a quadrilateral with one pair of parallel sides as described.**

**See Problem 2.**

**11.** The sides have length $a$ and $b$.

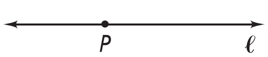
**12.** The sides have length $2a$ and $b$.

**13.** The sides have length $a$ and $\frac{1}{2}b$.

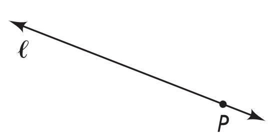
**For Exercises 14 and 15, draw a figure like the given one. Then construct the line that is perpendicular to $\ell$ at point $P$.**

**See Problem 3.**

**14.**

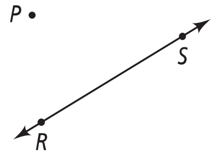



**15.**

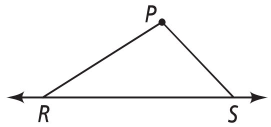


For Exercises 16–18, draw a figure like the given one. Then construct the line through point $P$ that is perpendicular to $\overleftrightarrow{RS}$. See Problem 4.

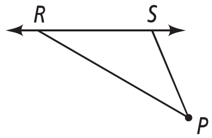
16.




17.



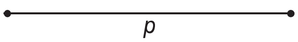

18.




**B Apply**

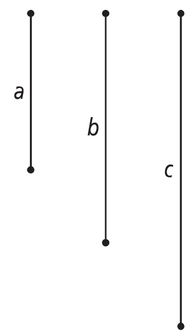
19. **Think About a Plan** Draw an acute angle. Construct an angle congruent to your angle so that the two angles are alternate interior angles.
- What does a sketch of the angle look like?
- Which construction(s) should you use?

20. **Constructions** Construct a square with side length $p$.




21. **Writing** Explain how to use the Converse of the Alternate Interior Angles Theorem to construct a line parallel to the given line through a point not on the line. (*Hint:* See Exercise 19.)

**For Exercises 22–28, use the segments at the right.**




22. Draw a line $m$. Construct a segment of length $b$ that is perpendicular to line $m$.

23. Construct a rectangle with base $b$ and height $c$.

24. Construct a square with sides of length $a$.

25. Construct a rectangle with one side of length $a$ and a diagonal of length $b$.

26. a. Construct a quadrilateral with a pair of parallel sides of length $c$.
b. **Make a Conjecture** What appears to be true about the other pair of sides in the quadrilateral you constructed?
c. Use a protractor, a ruler, or both to check the conjecture you made in part (b).

27. Construct a right triangle with legs of lengths $a$ and $b$.

28. a. Construct a triangle with sides of lengths $a$, $b$, and $c$.
b. Construct the midpoint of each side of the triangle.
c. Form a new triangle by connecting the midpoints.
d. **Make a Conjecture** How do the sides of the smaller triangle and the sides of the larger triangle appear to be related?
e. Use a protractor, ruler, or both to check the conjecture you made in part (d).

29. **Constructions** The diagrams below show steps for a parallel line construction.


a. List the construction steps in the correct order.
b. For the steps that use a compass, describe the location(s) of the compass point.

## Challenge

**Draw $\overline{DG}$. Construct a quadrilateral with diagonals that are congruent to $\overline{DG}$, bisect each other, and meet the given conditions. Describe the figure.**

**30.** The diagonals are not perpendicular.

**31.** The diagonals are perpendicular.

**Construct a rectangle with side lengths *a* and *b* that meets the given condition.**

**32.** $b = 2a$

**33.** $b = \frac{1}{2}a$

**34.** $b = \frac{1}{3}a$

**35.** $b = \frac{2}{3}a$

**Construct a triangle with side lengths *a*, *b*, and *c* that meets the given conditions. If such a triangle is not possible, explain.**

**36.** $a = b = c$

**37.** $a = b = 2c$

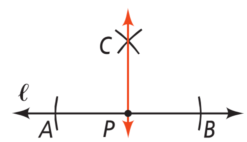
**38.** $a = 2b = 2c$

**39.** $a = b + c$

## Standardized Test Prep

**SAT/ACT**

**40.** The diagram at the right shows the construction of $\overleftrightarrow{CP}$ perpendicular to line $\ell$ through point *P*. Which of the following *must* be true?




(A) $\overleftrightarrow{CB} \parallel \overleftrightarrow{AB}$

(B) $CP = \frac{1}{2}AB$

(C) $\overleftrightarrow{AC} \parallel \overleftrightarrow{CB}$

(D) $\overline{AC} \cong \overline{BC}$

**41.** Suppose you construct lines $\ell$, *m*, and *n* so that $\ell \perp m$ and $\ell \parallel n$. Which of the following is true?

(F) $m \parallel n$

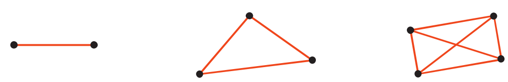
(G) $m \parallel \ell$

(H) $n \perp \ell$

(I) $n \perp m$

**Short Response**

**42.** For any two points, you can draw one segment. For any three noncollinear points, you can draw three segments. For any four noncollinear points, you can draw six segments. How many segments can you draw for eight noncollinear points? Explain your reasoning.

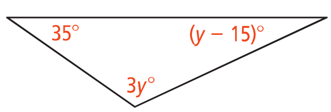


## Mixed Review

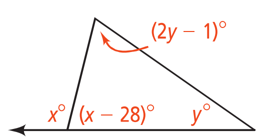
**Find each missing angle measure.**

See Lesson 3-5.

**43.**




**44.**




**Get Ready!** **To prepare for Lesson 3-7, do Exercises 45–47.**

**Simplify each ratio.**

See p. 831.

**45.** $\frac{2 - (-3)}{6 - (-4)}$

**46.** $\frac{1 - 4}{-2 - 1}$

**47.** $\frac{12 - 6}{2 - 5}$