5.6 Extension: Indirect Proof

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The indirect proof or proof by contradiction is a part of 41 out of 50 states' mathematic standards. Depending on the state, the teacher may choose to use none, part or all of this section.

Learning Objectives

- Reason indirectly to develop proofs.

Until now, we have proved theorems true by direct reasoning, where conclusions are drawn from a series of facts and previously proven theorems. However, we cannot always use direct reasoning to prove every theorem.

Indirect Proof: When the conclusion from a hypothesis is assumed false (or opposite of what it states) and then a contradiction is reached from the given or deduced statements.

The easiest way to understand indirect proofs is by example. You may choose to use the two-column format or a paragraph proof. First we will explore indirect proofs with algebra and then geometry.

Indirect Proofs in Algebra

Example 1: If $x = 2$, then $3x - 5 \neq 10$. Prove this statement is true by contradiction.

Solution: In an indirect proof the first thing you do is assume the conclusion of the statement is false. In this case, we will assume the opposite of $3x - 5 \neq 10$

If $x = 2$, then $3x - 5 = 10$

Now, proceed with this statement, as if it is true. Solve for x .

$$\begin{aligned} 3x - 5 &= 10 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

$x = 5$ contradicts the given statement that $x = 2$. Hence, our assumption is incorrect and $3x - 5$ cannot equal 10.

Example 2: If n is an integer and n^2 is odd, then n is odd. Prove this is true indirectly.

Solution: First, assume the opposite of "n is odd."

n is even.

Now, square n and see what happens.

If n is even, then $n = 2a$, where a is any integer.

$$n^2 = (2a)^2 = 4a^2$$

This means that n^2 is a multiple of 4. No odd number can be divided evenly by an even number, so this contradicts our assumption that n is even. Therefore, n must be odd if n^2 is odd.

Indirect Proofs in Geometry

Example 3: If $\triangle ABC$ is isosceles, then the measure of the base angles cannot be 92°. Prove this indirectly.

Solution: Assume the opposite of the conclusion.

The measure of the base angles is 92°.

If the base angles are 92°, then they add up to 184°. This contradicts the Triangle Sum Theorem that says all triangles add up to 180°. Therefore, the base angles cannot be 92°.

Example 4: Prove the SSS Inequality Theorem is true by contradiction.

Solution: The SSS Inequality Theorem says: "If two sides of a triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is greater in measure than the included angle of the second triangle." First, assume the opposite of the conclusion.

The included angle of the first triangle is less than or equal to the included angle of the second triangle.

If the included angles are equal then the two triangles would be congruent by SAS and the third sides would be congruent by CPCTC. This contradicts the hypothesis of the original statement "the third side of the first triangle is longer than the third side of the second." Therefore, the included angle of the first triangle must be larger than the included angle of the second.

To summarize:

- Assume the ***opposite*** of the conclusion (second half) of the statement.
- Proceed as if this assumption is true to find the ***contradiction***.

- Once there is a contradiction, the original statement is true.
- ***DO NOT use specific examples.*** Use variables so that the contradiction can be generalized.

Review Questions

Prove the following statements true indirectly.

1. If n is an integer and n^2 is even, then n is even.
2. If $m\angle A \neq m\angle B$ in $\triangle ABC$, then $\triangle ABC$ is not equilateral.
3. If $x > 3$, then $x^2 > 9$.
4. The base angles of an isosceles triangle are congruent.
5. If x is even and y is odd, then $x + y$ is odd.
6. In $\triangle ABE$, if $\angle A$ is a right angle, then $\angle B$ cannot be obtuse.
7. If A, B, and C are collinear, then $AB + BC = AC$ (Segment Addition Postulate).
8. If a collection of nickels and dimes is worth 85 cents, then there must be an odd number of nickels.
9. Hugo is taking a true/false test in his Geometry class. There are five questions on the quiz. The teacher gives her students the following clues: The last answer on the quiz is not the same as the fourth answer. The third answer is true. If the fourth answer is true, then the one before it is false. Use an indirect proof to prove that the last answer on the quiz is true.
10. On a test of 15 questions, Charlie claims that his friend Suzie must have gotten at least 10 questions right. Another friend, Larry, does not agree and suggests that Suzie could not have gotten that many correct. Rebecca claims that Suzie certainly got at least one question correct. If *only one* of these statements is true, how many questions did Suzie get right?