1.4 Midpoints and Bisectors

Difficulty Level: **At Grade** | Created by: CK-12
Last Modified: Dec 25, 2014

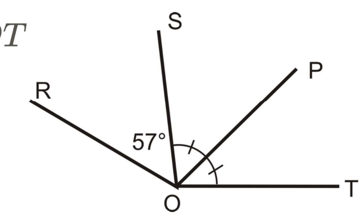
Learning Objectives

- Identify the midpoint of line segments.
- Identify the bisector of a line segment.
- Understand and the Angle Bisector Postulate.

Review Queue

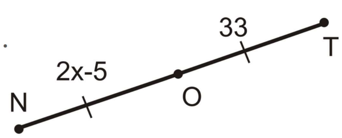
Answer the following questions.

$m\angle ROT = 165^\circ$, find $m\angle POT$



[Figure 1]

Find x .



[Figure 2]

1. Use the Angle Addition Postulate to write an equation for the angles in #1.

Know What? The building below is the Transamerica Pyramid in San Francisco. This building was completed in 1972 and, at that time was one of the tallest buildings in the world. It is a pyramid with two "wings" on either side, to accommodate elevators. Because San Francisco has problems with earthquakes, there are regulations on how a building can be designed. In order to make this building as tall as it is and still abide by the building codes, the designer used this pyramid shape.

It is very important in designing buildings that the angles and parts of the building are equal. What components of this building look equal? Analyze angles, windows, and the sides of the building.

[Figure 3]

Congruence

You could argue that another word for ***equal*** is ***congruent***. However, the two differ slightly.

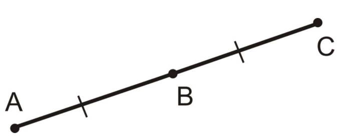
Congruent: When two geometric figures have the same shape and size.

We label congruence with a $\cong$ sign. Notice the $\sim$ above the = sign. $\overline{AB} \cong \overline{BA}$ means that $\overline{AB}$ is congruent to $\overline{BA}$. If we know two segments or angles are congruent, then their measures are also equal. If two segments or angles have the same measure, then, they are also congruent.

Equal	***Congruent***
$=$	$\cong$
used with ***measurement***	used with ***figures***
$m\overline{AB} = AB = 5\ cm$	$\overline{AB} \cong \overline{BA}$
$m\angle ABC = 60^\circ$	$\angle ABC \cong \angle CBA$

Midpoints

Midpoint: A point on a line segment that divides it into two congruent segments.



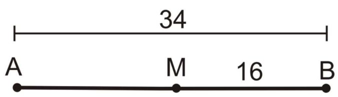
[Figure 4]

Because $AB = BC$, B is the midpoint of $\overline{AC}$.

Midpoint Postulate: Any line segment will have exactly one midpoint.

This might seem self-explanatory. However, be careful, this postulate is referring to the *midpoint,* not the lines that pass through the midpoint, which is infinitely many.

Example 1: Is M a midpoint of $\overline{AB}$?

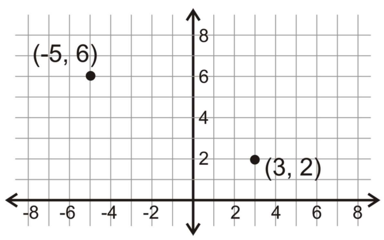


[Figure 5]

Solution: No, it is not because $MB = 16$ and $AM = 34 - 16 = 18$.

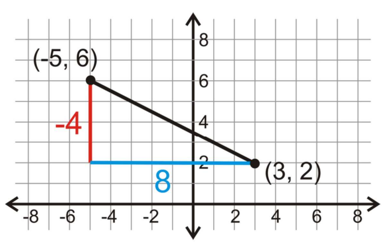
Midpoint Formula

When points are plotted in the coordinate plane, you can use slope to find the midpoint between then. We will generate a formula here.



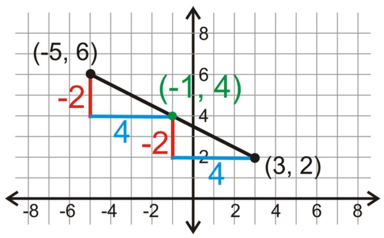
[Figure 6]

Here are two points, (-5, 6) and (3, 4). Draw a line between the two points and determine the vertical distance and the horizontal distance.



[Figure 7]

So, it follows that the midpoint is down and over half of each distance. The midpoint would then be down 2 (or -2) from (-5, 6) and over positively 4. If we do that we find that the midpoint is (-1, 4).



[Figure 8]

Let's create a formula from this. If the two endpoints are (-5, 6) and (3, 4), then the midpoint is (-1, 4). -1 is ***halfway*** between -5 and 3 and 4 is ***halfway*** between 6 and 2. Therefore, the formula for the midpoint is the average of the $x-$ values and the average of the $y-$ values.

Midpoint Formula: For two points, $(x_1,\ y_1)$ and $(x_2,\ y_2)$, the midpoint is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example 2: Find the midpoint between (9, -2) and (-5, 14).

Solution: Plug the points into the formula.

$$\left(\frac{9+(-5)}{2}, \frac{-2+14}{2}\right)=\left(\frac{4}{2}, \frac{12}{2}\right)=(2,6)$$

Example 3: If $M(3,-1)$ is the midpoint of $\overline{AB}$ and $B(7,-6)$, find A.

Solution: Plug what you know into the midpoint formula.

$$\begin{aligned}\left(\frac{7+x_A}{2}, \frac{-6+y_A}{2}\right) &= (3,-1)\\ \frac{7+x_A}{2} &= 3 \text{ and } \frac{-6+y_A}{2}=-1 \qquad A \text{ is } (-1,4).\\ 7+x_A &= 6 \text{ and } -6+y_A=-2\\ x_A &= -1 \text{ and } y_A=4\end{aligned}$$

Another way to find the other endpoint is to find the difference between M and B and then duplicate it on the other side of M.

$x-$ *values*: $7-3=4$, so 4 on the other side of 3 is $3-4=-1$

$y-$ *values*: $-6-(-1)=-5$, so -5 on the other side of -1 is $-1-(-5)=4$

A is still (-1, 4). You may use either method.

Segment Bisectors

Segment Bisector: A line, segment, or ray that passes through a midpoint of another segment.

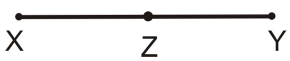
A bisector cuts a line segment into two congruent parts.

Example 4: Use a ruler to draw a bisector of the segment below.

[Figure 9]

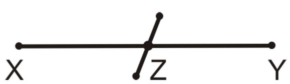
Solution: The first step in identifying a bisector is finding the midpoint. Measure the line segment and it is 4 cm long. To find the midpoint, divide 4 by 2.

So, the midpoint will be 2 cm from either endpoint, or halfway between. Measure 2 cm from one endpoint and draw the midpoint.



[Figure 10]

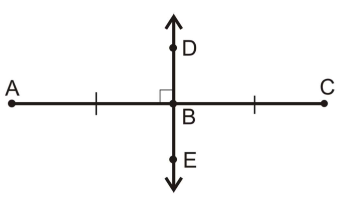
To finish, draw a line that passes through the midpoint. It doesn't matter how the line intersects $\overline{XY}$, as long as it passes through Z.



[Figure 11]

A specific type of segment bisector is called a perpendicular bisector.

Perpendicular Bisector: A line, ray or segment that passes through the midpoint of another segment and intersects the segment at a right angle.



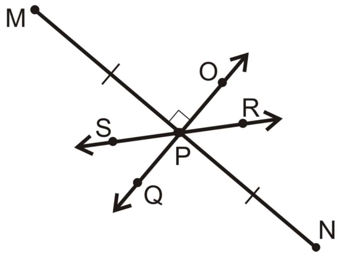
[Figure 12]

$\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AC}$, so $\overline{AB} \cong \overline{BC}$ and $\overline{AC} \perp \overleftrightarrow{DE}$.

Perpendicular Bisector Postulate: For every line segment, there is one perpendicular bisector that passes through the midpoint.

There are *infinitely many bisectors*, but only *one perpendicular bisector* for any segment.

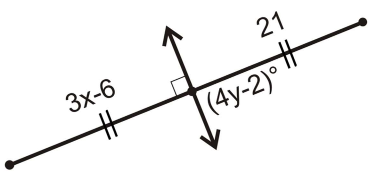
Example 5: Which line is the perpendicular bisector of $\overline{MN}$?



[Figure 13]

Solution: The perpendicular bisector must bisect $\overline{MN}$ and be perpendicular to it. Only $\overleftrightarrow{OQ}$ satisfies both requirements. $\overleftrightarrow{SR}$ is just a bisector.

Example 6: ***Algebra Connection*** Find x and y.



[Figure 14]

Solution: The line shown is the perpendicular bisector. So, $3x-6=21,\ 3x=27,\ x=9$.

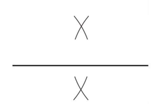
And, $(4y-2)^\circ = 90^\circ, 4y^\circ = 92^\circ, y = 23^\circ$.

Investigation 1-3: Constructing a Perpendicular Bisector

Draw a line that is at least 6 cm long, about halfway down your page.

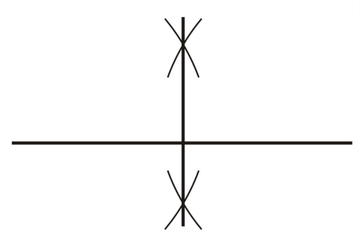
[Figure 15]

Place the pointer of the compass at an endpoint. Open the compass to be greater than half of the segment. Make arc marks above and below the segment. Repeat on the other endpoint. Make sure the arc marks intersect.



[Figure 16]

Use your straight edge to draw a line connecting the arc intersections.

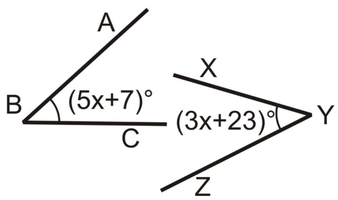


[Figure 17]

This constructed line bisects the line you drew in #1 and intersects it at 90°. So, this construction also works to create a right angle. To see an animation of this investigation, go to http://www.mathsisfun.com/geometry/construct-linebisect.html.

Congruent Angles

Example 7: *Algebra Connection* What is the measure of each angle?



[Figure 18]

Solution: From the picture, we see that the angles are congruent, so the given measures are equal.

$$(5x+7)^\circ = (3x+23)^\circ$$
$$2x^\circ = 16^\circ$$
$$x = 8^\circ$$

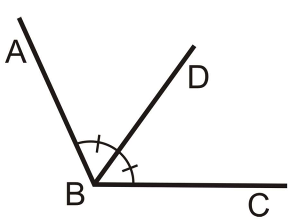
To find the measure of $\angle ABC$, plug in $x = 8^\circ$ to $(5x+7)^\circ$.

$$(5(8)+7)^\circ$$
$$(40+7)^\circ$$
$$47^\circ$$

Because $m\angle ABC = m\angle XYZ,\ m\angle XYZ = 47^\circ$ too.

Angle Bisectors

Angle Bisector: A ray that divides an angle into two congruent angles, each having a measure exactly half of the original angle.



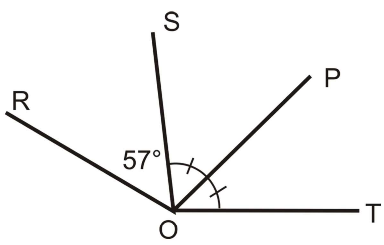
[Figure 19]

$\overline{BD}$ is the angle bisector of $\angle ABC$

$$\angle ABD \cong \angle DBC$$
$$m\angle ABD = \frac{1}{2} m\angle ABC$$

Angle Bisector Postulate: Every angle has exactly one angle bisector.

Example 8: Let's take a look at Review Queue #1 again. Is $\overline{OP}$ the angle bisector of $\angle SOT$? Recall, that $m\angle ROT = 165^\circ$, what is $m\angle SOP$ and $m\angle POT$?

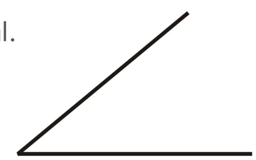


[Figure 20]

Solution: Yes, $\overline{OP}$ is the angle bisector of $\angle SOT$ according to the markings in the picture. If $m\angle ROT = 165^\circ$ and $m\angle ROS = 57^\circ$, then $m\angle SOT = 165^\circ - 57^\circ = 108^\circ$. The $m\angle SOP$ and $m\angle POT$ are each half of 108° or 54° .

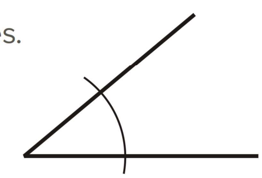
Investigation 1-4: Constructing an Angle Bisector

Draw an angle on your paper. Make sure one side is horizontal.



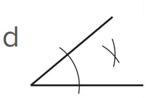
[Figure 21]

Place the pointer on the vertex. Draw an arc that intersects both sides.



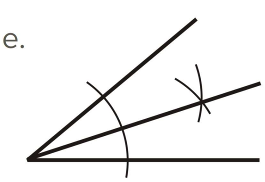
[Figure 22]

Move the pointer to the arc intersection with the horizontal side. Make a second arc mark on the interior of the angle. Repeat on the other side. Make sure they intersect.



[Figure 23]

Connect the arc intersections from #3 with the vertex of the angle.



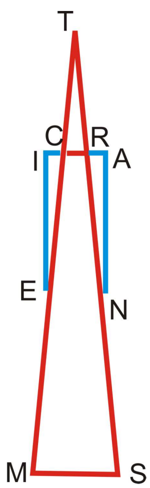
[Figure 24]

To see an animation of this construction, view
http://www.mathsisfun.com/geometry/construct-anglebisect.html.

Know What? Revisited The image to the right is an outline of the Transamerica Building from earlier in the lesson. From this outline, we can see the following parts are congruent:

$$\begin{aligned}
\overline{TR} &\cong \overline{TC} & & \angle TCR \cong \angle TRC \\
\overline{RS} &\cong \overline{CM} & & \angle CIE \cong \angle RAN \\
\overline{CI} &\cong \overline{RA} & \text{and} \quad & \angle TMS \cong \angle TSM \\
\overline{AN} &\cong \overline{IE} & & \angle IEC \cong \angle ANR \\
\overline{TS} &\cong \overline{TM} & & \angle TCI \cong \angle TRA
\end{aligned}$$

As well at these components, there are certain windows that are congruent and all four triangular sides of the building are congruent to each other.

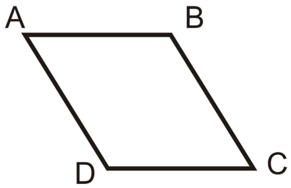


[Figure 25]

Review Questions

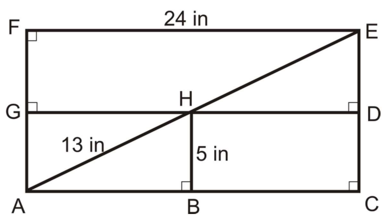
1. Copy the figure below and label it with the following information:

$$\begin{aligned}
\angle A &\cong \angle C \\
\angle B &\cong \angle D \\
\overline{AB} &\cong \overline{CD} \\
\overline{AD} &\cong \overline{BC}
\end{aligned}$$



[Figure 26]

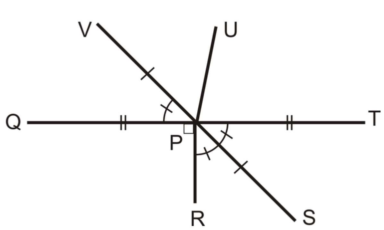
For 2-9, find the lengths, given: H is the midpoint of $\overline{AE}$ and $\overline{DG}$, B is the midpoint of $\overline{AC}$, $\overline{GD}$ is the perpendicular bisector of $\overline{FA}$ and $\overline{EC}$, $\overline{AC} \cong \overline{FE}$, and $\overline{FA} \cong \overline{EC}$.



[Figure 27]

2. AB
3. GA
4. ED
5. HE
6. $m\angle HDC$
7. FA
8. GD
9. $m\angle FED$
10. How many copies of triangle AHB can fit inside rectangle $FECA$ without overlapping?

For 11-18, use the following picture to answer the questions.

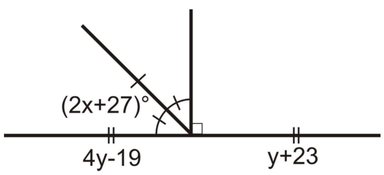


[Figure 28]

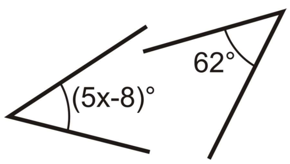
11. What is the angle bisector of $\angle TPR$?
12. P is the midpoint of what two segments?
13. What is $m\angle QPR$?

14. What is $m\angle TPS$?
15. How does $\overline{VS}$ relate to $\overline{QT}$?
16. How does $\overline{QT}$ relate to $\overline{VS}$?
17. Is $\overline{PU}$ a bisector? If so, of what?
18. What is $m\angle QPV$?

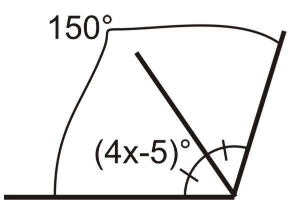
Algebra Connection For 19-24, use algebra to determine the value of variable(s) in each problem.



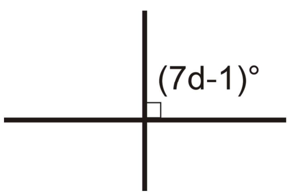
[Figure 29]



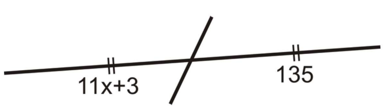
[Figure 30]



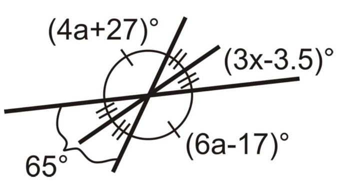
[Figure 31]



[Figure 32]



[Figure 33]



[Figure 34]

19. ***Construction*** Using your protractor, draw an angle that is 110° . Then, use your compass to construct the angle bisector. What is the measure of each angle?
20. ***Construction*** Using your protractor, draw an angle that is 75° . Then, use your compass to construct the angle bisector. What is the measure of each angle?
21. ***Construction*** Using your ruler, draw a line segment that is 7 cm long. Then use your compass to construct the perpendicular bisector, What is the measure of each segment?
22. ***Construction*** Using your ruler, draw a line segment that is 4 in long. Then use your compass to construct the perpendicular bisector, What is the measure of each segment?
23. ***Construction*** Draw a straight angle (180°) . Then, use your compass to construct the angle bisector. What kind of angle did you just construct?

For questions 30-33, find the midpoint between each pair of points.

30. (-2, -3) and (8, -7)
31. (9, -1) and (-6, -11)
32. (-4, 10) and (14, 0)
33. (0, -5) and (-9, 9)

Given the midpoint (M) and either endpoint of $\overline{AB}$, find the other endpoint.

34. $A(-1,2)$ and $M(3,6)$
35. $B(-10,-7)$ and $M(-2,1)$
36. ***Error Analysis*** Erica is looking at a geometric figure and trying to determine which parts are congruent. She wrote $\overline{AB}=\overline{CD}$. Is this correct? Why or why not?
37. ***Challenge*** Use the Midpoint Formula to solve for the $x-$ value of the midpoint and the $y-$ value of the midpoint. Then, use this formula to solve #34. Do you get the same answer?
38. ***Construction Challenge*** Use construction tools and the constructions you have learned in this section to construct a 45° angle.
39. ***Construction Challenge*** Use construction tools and the constructions you have learned in this section to construct two 2 in segments that bisect each other. Now connect all four endpoints with segments. What figure have you constructed?
40. Describe an example of how the concept of midpoint (or the midpoint formula) could be used in the real world.

Review Queue Answers

1. See Example 6
2. $$\begin{aligned} 2x - 5 &= 33 \\ 2x &= 38 \\ x &= 19 \end{aligned}$$
3. $m\angle ROT = m\angle ROS + m\angle SOP + m\angle POT$

1.5 Basic Constructions

Difficulty Level: **At Grade** | Created by: CK-12

Last Modified: Feb 18, 2016

To Construct Bisector of an Angle

- http://www.mathsisfun.com/geometry/construct-anglebisect.html

To construct Perpendicular Bisector of a Line Segment

Check this link below

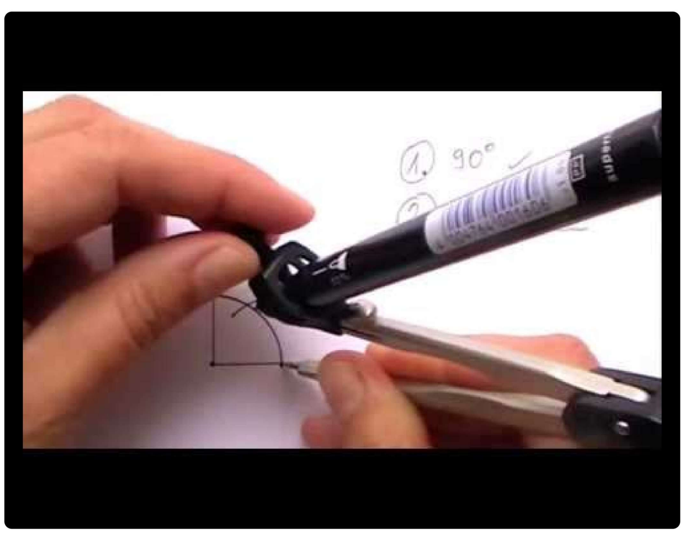
- http://www.mathopenref.com/constbisectline.html

Construction of Angles

60 Degrees: http://www.mathsisfun.com/geometry/construct-60degree.html

30 Degrees: http://www.mathsisfun.com/geometry/construct-30degree.html

45 Degrees: https://www.youtube.com/watch?v=QUtSDBQV1UA



https://www.ck12.org/flx/render/embeddedobject/162681


[Figure 2]

This bridge was designed so that $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$. Are the support beams parallel?

Corresponding Angles Converse

Recall that the converse of a statement switches the conclusion and the hypothesis. So, if a, then b becomes if b, then a. We will find the converse of all the theorems from the last section and will determine if they are true.

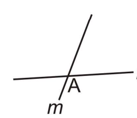
The Corresponding Angles Postulate says: *If two lines are parallel, then the corresponding angles are congruent.* The converse is:

Converse of Corresponding Angles Postulate: If corresponding angles are congruent when two lines are cut by a transversal, then the lines are parallel.

Is this true? For example, if the corresponding angles both measured 60°, would the lines be parallel? YES. All eight angles created by l, m and the transversal are either 60° or 120°, making the slopes of l and m the same which makes them parallel. This can also be seen by using a construction.

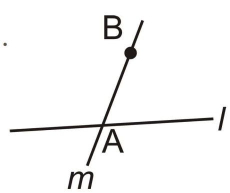
Investigation 3-5: Creating Parallel Lines using Corresponding Angles

Draw two intersecting lines. Make sure they are not perpendicular. Label them l and m, and the point of intersection, A, as shown.



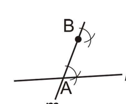
[Figure 3]

Create a point, B, on line m, above A.



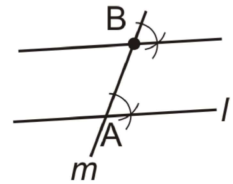
[Figure 4]

Copy the acute angle at A (the angle to the right of m) at point B. See Investigation 2-2 in Chapter 2 for the directions on how to copy an angle.



[Figure 5]

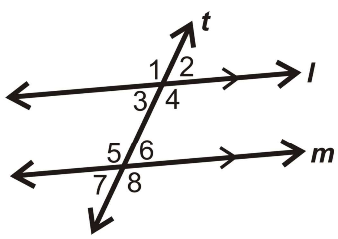
Draw the line from the arc intersections to point B.



[Figure 6]

From this construction, we can see that the lines are parallel.

Example 1: If $m\angle 8 = 110^\circ$ and $m\angle 4 = 110^\circ$, then what do we know about lines l and m?



[Figure 7]

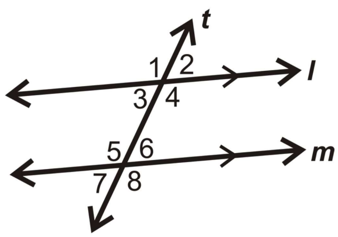
Solution: $\angle 8$ and $\angle 4$ are corresponding angles. Since $m\angle 8 = m\angle 4$, we can conclude that $l \,||\, m$.

Alternate Interior Angles Converse

We also know, from the last lesson, that when parallel lines are cut by a transversal, the alternate interior angles are congruent. The converse of this theorem is also true:

Converse of Alternate Interior Angles Theorem: If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

Example 3: Prove the Converse of the Alternate Interior Angles Theorem.



[Figure 8]

Given: l and m and transversal t

$\angle 3 \cong \angle 6$

Prove: $l \,||\, m$

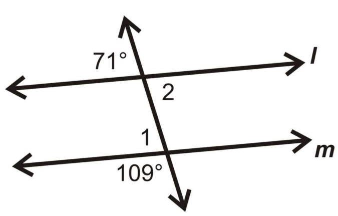
Solution:

Statement	*Reason*
1. l and m and transversal $t \angle 3 \cong \angle 6$	Given
2. $\angle 3 \cong \angle 2$	Vertical Angles Theorem
3. $\angle 2 \cong \angle 6$	Transitive PoC
4. $l \,||\, m$	Converse of the Corresponding Angles Postulate

Prove Move: Shorten the names of these theorems. Discuss with your teacher an appropriate abbreviations. For example, the Converse of the Corresponding Angles Theorem could be "Converse CA Thm" or "ConvCA."

Notice that the Corresponding Angles Postulate was not used in this proof. The Transitive Property is the reason for Step 3 because we do not know if l is parallel to m until we are done with the proof. You could conclude that if we are trying to prove two lines are parallel, the converse theorems will be used. And, if we are proving two angles are congruent, we must be given that the two lines are parallel.

Example 4: Is $l \,||\, m$?

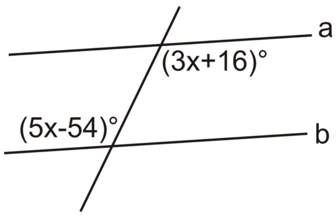


[Figure 9]

Solution: First, find $m\angle 1$. We know its linear pair is 109°. By the Linear Pair Postulate, these two angles add up to 180°, so $m\angle 1 = 180^\circ - 109^\circ = 71^\circ$. This means that $l \,||\, m$, by the Converse of the Corresponding Angles Postulate.

Example 5: ***Algebra Connection*** What does x have to be to make $a \,||\, b$?

Solution: Because these are alternate interior angles, they must be equal for $a \,||\, b$. Set the expressions equal to each other and solve.



[Figure 10]

$$\begin{aligned}3x+16^\circ &= 5x-54^\circ\\ 70^\circ &= 2x\\ 35^\circ &= x \qquad \text{To make } a \,||\, b,\ x=35^\circ.\end{aligned}$$

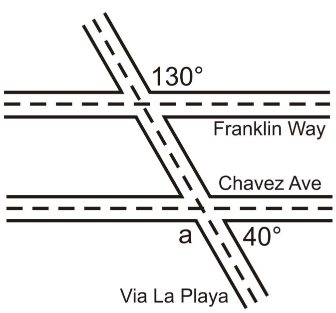
Converse of Alternate Exterior Angles & Consecutive Interior Angles

You have probably guessed that the converse of the Alternate Exterior Angles Theorem and the Consecutive Interior Angles Theorem areal so true.

Converse of the Alternate Exterior Angles Theorem: If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

Example 6: *Real-World Situation* The map below shows three roads in Julio's town.

Julio used a surveying tool to measure two angles at the intersections in this picture he drew (NOT to scale). ***Julio wants to know if Franklin Way is parallel to Chavez Avenue.***



[Figure 11]

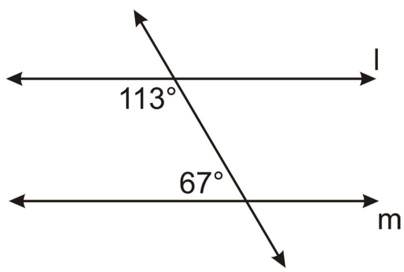
Solution: The labeled 130° angle and $\angle a$ are alternate exterior angles. If $m\angle a = 130^\circ$, then the lines are parallel. To find $m\angle a$, use the other labeled angle which is 40°, and its linear pair. Therefore, $\angle a + 40^\circ = 180^\circ$ and $\angle a = 140^\circ$. $140^\circ \neq 130^\circ$, so Franklin Way and Chavez Avenue are not parallel streets.

The final converse theorem is of the Same Side Interior Angles Theorem. Remember that these angles are not congruent when lines are parallel, they are **supplementary**.

Converse of the Same Side Interior Angles Theorem: If two lines are cut by a transversal and the consecutive interior angles are supplementary, then the lines are parallel.

Example 7: Is $l || m$? How do you know?

Solution: These are Same Side Interior Angles. So, if they add up to 180°, then $l || m$. $113^\circ + 67^\circ = 180^\circ$, therefore $l || m$.



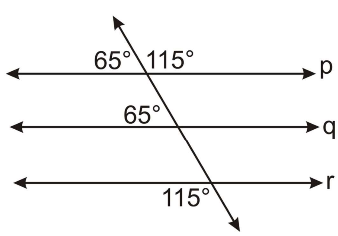
[Figure 12]

Parallel Lines Property

The Parallel Lines Property is a transitive property that can be applied to parallel lines. Remember the Transitive Property of Equality is: If $a = b$ and $b = c$, then $a = c$. The Parallel Lines Property changes = to $||$.

Parallel Lines Property: If lines $l || m$ and $m || n$, then $l || n$.

Example 8: Are lines q and r parallel?



[Figure 13]

Solution: First find if $p || q$, followed by $p || r$. If so, then $q || r$.

$p || q$ by the Converse of the Corresponding Angles Postulate, the corresponding angles are 65°. $p || r$ by the Converse of the Alternate Exterior Angles Theorem, the alternate exterior angles are 115°. Therefore, by the Parallel Lines Property, $q || r$.

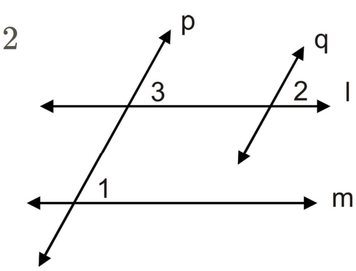
Know What? Revisited: The CoronadoBridge has $\angle 1$ and $\angle 2$, which are corresponding angles. These angles must be equal for the beams to be parallel. $\angle 1 = 92^\circ$ and $\angle 2 = 88^\circ$ and $92^\circ \neq 88^\circ$, so the beams are not parallel, therefore a sturdy and safe girder bridge.

Review Questions

1. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing congruent alternate interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy the angle in a different location.
2. ***Construction*** Using Investigation 3-1 to help you, show that two lines are parallel by constructing supplementary consecutive interior angles. HINT: Steps 1 and 2 will be exactly the same, but at step 3, you will copy a different angle.

For Questions 3-5, fill in the blanks in the proofs below.

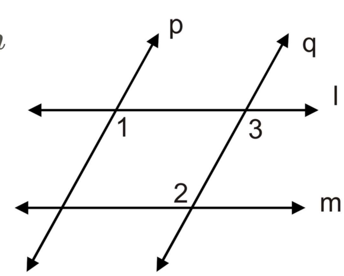
Given: $l \, || \, m, \; p \, || \, q$ Prove: $\angle 1 \cong \angle 2$



[Figure 14]

Statement	*Reason*
1. $l \, \| \, m$	1.
2.	2. Corresponding Angles Postulate
3. $p \, \| \, q$	3.
4.	4.
5. $\angle 1 \cong \angle 2$	5.

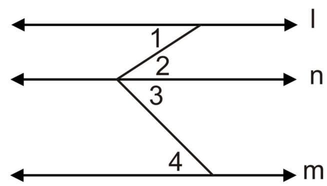
Given: $p \, || \, q, \; \angle 1 \cong \angle 2$ Prove: $l \, || \, m$



[Figure 15]

Statement	Reason
1. $p \mid\mid q$	1.
2.	2. Corresponding Angles Postulate
3. $\angle 1 \cong \angle 2$	3.
4.	4. Transitive PoC
5.	5. Converse of Alternate Interior Angles Theorem

Given: $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$ Prove: $l \mid\mid m$

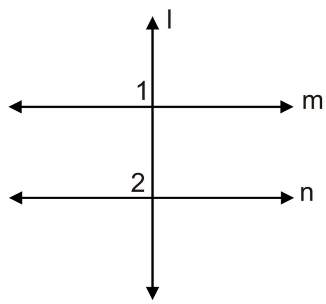


[Figure 16]

Statement	Reason
1. $\angle 1 \cong \angle 2$	1.
2. $l \mid\mid n$	2.
3. $\angle 3 \cong \angle 4$	3.
4.	4. Converse of Alternate Interior Angles Theorem
5. $l \mid\mid m$	5.

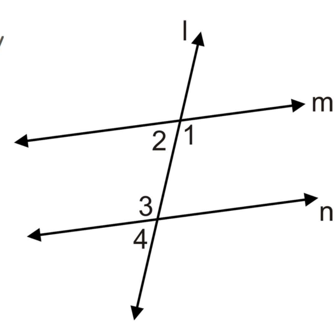
For Questions 6-9, create your own two column proof.

Given: $m \perp l,\ n \perp l$ Prove: $m \mid\mid n$



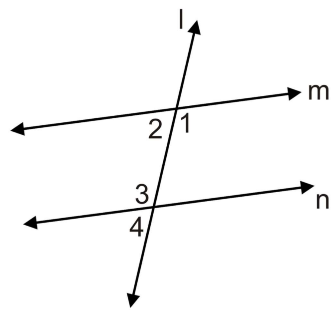
[Figure 17]

Given: $\angle 1 \cong \angle 3$ Prove: $\angle 1$ and $\angle 4$ are supplementary



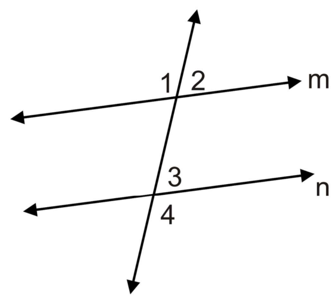
[Figure 18]

Given: $\angle 2 \cong \angle 4$ Prove: $\angle 1 \cong \angle 3$



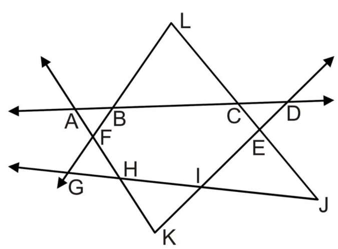
[Figure 19]

Given: $\angle 2 \cong \angle 3$ Prove: $\angle 1 \cong \angle 4$



[Figure 20]

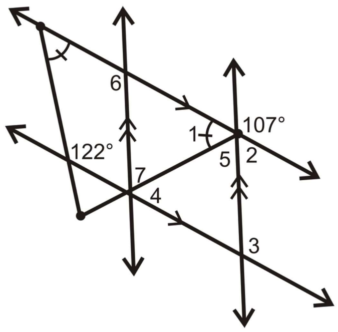
In 10-15, use the given information to determine which lines are parallel. If there are none, write *none*. Consider each question individually.



[Figure 21]

10. $\angle LCD \cong \angle CJI$
11. $\angle BCE \, and \, \angle BAF$ are supplementary
12. $\angle FGH \cong \angle EIJ$
13. $\angle BFH \cong \angle CEI$
14. $\angle LBA \cong \angle IHK$
15. $\angle ABG \cong \angle BGH$

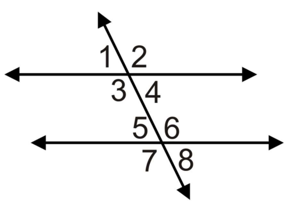
In 16-22, find the measure of the lettered angles below.



[Figure 22]

16. $m\angle 1$
17. $m\angle 2$
18. $m\angle 3$
19. $m\angle 4$
20. $m\angle 5$
21. $m\angle 6$
22. $m\angle 7$

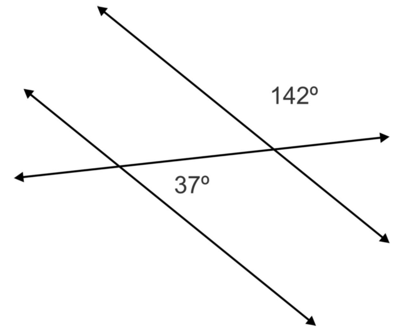
For 23-27, what does x have to measure to make the lines parallel?



[Figure 23]

23. $m\angle 3 = (3x + 25)^\circ$ and $m\angle 5 = (4x - 55)^\circ$
24. $m\angle 2 = (8x)^\circ$ and $m\angle 7 = (11x - 36)^\circ$
25. $m\angle 1 = (6x - 5)^\circ$ and $m\angle 5 = (5x + 7)^\circ$
26. $m\angle 4 = (3x - 7)^\circ$ and $m\angle 7 = (5x - 21)^\circ$
27. $m\angle 1 = (9x)^\circ$ and $m\angle 6 = (37x)^\circ$
28. ***Construction*** Draw a straight line. Construct a line perpendicular to this line through a point on the line. Now, construct a perpendicular line to this new line. What can you conclude about the original line and this final line?
29. How could you prove your conjecture from problem 28?

What is wrong in the following diagram, given that $j \| k$?



[Figure 24]

Review Queue Answers

1. Answers:

   a. *If I am out of school, then it is summer.*

   b. *If I go to the mall, then I am done with my homework.*

   c. *If corresponding angles created by two lines cut by a transversal are congruent, then the two lines are parallel.*

2. Answers:

   a. Not true, I could be out of school on any school holiday or weekend during the school year.

   b. Not true, I don't have to be done with my homework to go to the mall.

   c. Yes, because if two corresponding angles are congruent, then the slopes of these two lines have to be the same, making the lines parallel.

3. The two angles are supplementary.

$$\begin{aligned}(17x+14)^\circ+(4x-2)^\circ &= 180^\circ\\ 21x+12^\circ &= 180^\circ\\ 21x &= 168^\circ\\ x &= 8^\circ\end{aligned}$$

3.4 Properties of Perpendicular Lines

Difficulty Level: **Basic** | Created by: CK-12
Last Modified: Dec 25, 2014

Learning Objectives

- Understand the properties of perpendicular lines.
- Explore problems with parallel lines and a perpendicular transversal.
- Solve problems involving complementary adjacent angles.

Review Queue

Determine if the following statements are true or false. If they are true, write the converse. If they are false, find a counter example.

1. Perpendicular lines form four right angles.

2. A right angle is greater than or equal to 90° .

Find the slope between the two given points.

3. (-3, 4) and (-3, 1)

4. (6, 7) and (-5, 7)

Know What? There are several examples of slope in nature. Below are pictures of Half Dome in Yosemite National Park and the horizon over the Pacific Ocean. These are examples of horizontal and vertical lines in real life. Can you determine the slope of these lines?

[Figure 1]

Congruent Linear Pairs