9.1 Parts of Circles & Tangent Lines

Last Modified: Dec 25, 2014

Learning Objectives

- Define circle, center, radius, diameter, chord, tangent, and secant of a circle.
- Explore the properties of tangent lines and circles.

Review Queue

1. Find the equation of the line with $m = -2$ and passes through (4, -5).
2. Find the equation of the line that passes though (6, 2) and (-3, -1).
3. Find the equation of the line *perpendicular* to the line in #2 and passes through (-8, 11).

Know What? The clock to the right is an ancient astronomical clock in Prague. It has a large background circle that tells the local time and the "ancient time" and then the smaller circle rotates around on the orange line to show the current astrological sign. The yellow point is the center of the larger clock. How does the orange line relate to the small and larger circle? How does the hand with the moon on it (black hand with the circle) relate to both circles? Are the circles concentric or tangent?

[Figure 1]

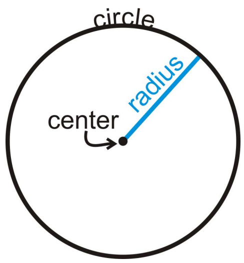
For more information on this clock, see: http://en.wikipedia.org/wiki/Prague_Astronomical_Clock

Defining Terms

Circle: The set of all points that are the same distance away from a specific point, called the **center.**

Radius: The distance from the center to the circle.

The center is typically labeled with a capital letter because it is a point. If the center is A, we would call this circle, "circle A," and labeled $\odot A$. Radii (the plural of radius) are line segments. There are infinitely many radii in any circle.

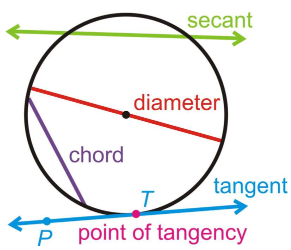


[Figure 2]

Chord: A line segment whose endpoints are on a circle.

Diameter: A chord that passes through the center of the circle.

Secant: A line that intersects a circle in two points.



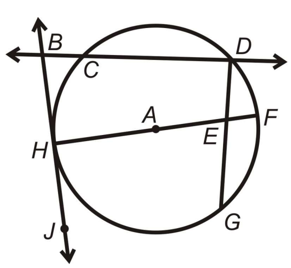
[Figure 3]

Tangent: A line that intersects a circle in exactly one point.

Point of Tangency: The point where the tangent line touches the circle.

Notice that the tangent ray $\overrightarrow{TP}$ and tangent segment $\overline{TP}$ are also called tangents. The length of a diameter is two times the length of a radius.

Example 1: Identify the parts of $\odot A$ that best fit each description.



[Figure 4]

a) A radius

b) A chord

c) A tangent line

d) The point of tangency

e) A diameter

f) A secant

Solution:

a) HA or AF

b) CD, HF , or DG

c) $\overleftrightarrow{BJ}$

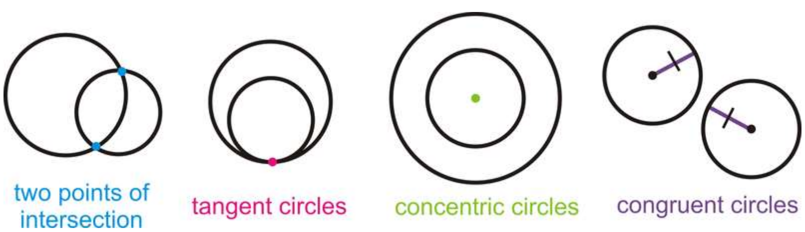
d) *Point* H

e) HF

f) $\overleftrightarrow{BD}$

Coplanar Circles

Two circles can intersect in two points, one point, or no points. If two circles intersect in one point, they are called *tangent circles.*



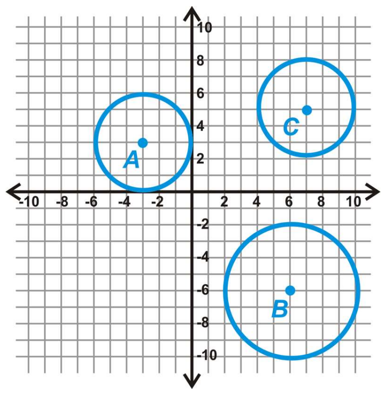
[Figure 5]

Congruent Circles: Two circles with the same radius, but different centers.

Concentric Circles: When two circles have the same center, but different radii.

If two circles have different radii, they are similar. *All circles are similar.*

Example 2: Determine if any of the following circles are congruent.



[Figure 6]

Solution: From each center, count the units to the circle. It is easiest to count vertically or horizontally. Doing this, we have:

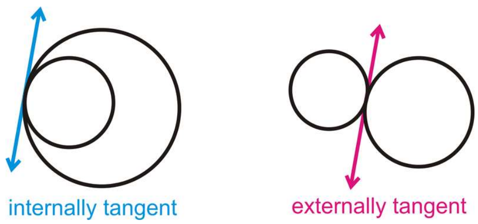
$$\text{Radius of } \odot A = 3\ units$$
$$\text{Radius of } \odot B = 4\ units$$
$$\text{Radius of } \odot C = 3\ units$$

From these measurements, we see that $\odot A \cong \odot C$.

Notice that two circles are congruent, just like two triangles or quadrilaterals. Only the *lengths* of the radii are equal.

Tangent Lines

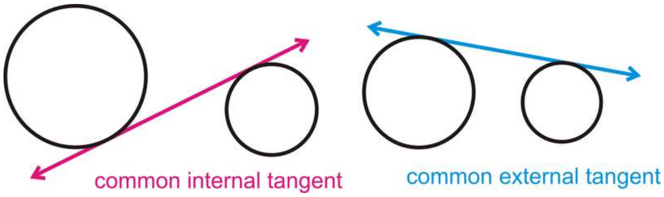
We just learned that two circles can be tangent to each other. Two triangles can be tangent in two different ways, either *internally* tangent or *externally* tangent.



[Figure 7]

If the circles are not tangent, they can share a tangent line, called a *common* tangent. Common tangents can be internally tangent and externally tangent too.

Notice that the common internal tangent passes through the space between the two circles. Common external tangents stay on the top or bottom of both circles.



[Figure 8]

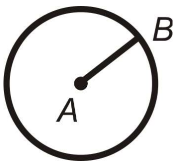
Tangents and Radii

The tangent line and the radius drawn to the point of tangency have a unique relationship. Let's investigate it here.

Investigation 9-1: Tangent Line and Radius Property

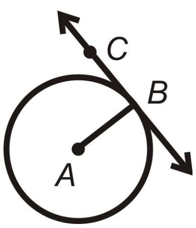
Tools needed: compass, ruler, pencil, paper, protractor

Using your compass, draw a circle. Locate the center and draw a radius. Label the radius AB, with A as the center.



[Figure 9]

Draw a tangent line, $\overleftrightarrow{BC}$, where B is the point of tangency. To draw a tangent line, take your ruler and line it up with point B. Make sure that B is the only point on the circle that the line passes through.



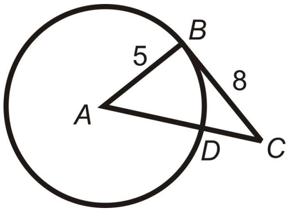
[Figure 10]

1. Using your protractor, find $m\angle ABC$.

Tangent to a Circle Theorem: A line is tangent to a circle if and only if the line is perpendicular to the radius drawn to the point of tangency.

To prove this theorem, the easiest way to do so is indirectly (proof by contradiction). Also, notice that this theorem uses the words "if and only if," making it a biconditional statement. Therefore, the converse of this theorem is also true.

Example 3: In $\odot A, CB$ is tangent at point B. Find AC. Reduce any radicals.



[Figure 11]

Solution: Because CB is tangent, $AB \perp CB$, making $\triangle ABC$ a right triangle. We can use the Pythagorean Theorem to find AC.

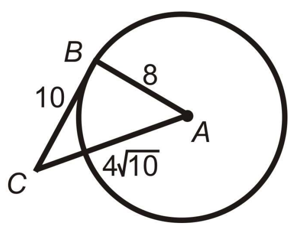
$$\begin{aligned} 5^2 + 8^2 &= AC^2 \\ 25 + 64 &= AC^2 \\ 89 &= AC^2 \\ AC &= \sqrt{89} \end{aligned}$$

Example 4: Find DC, in $\odot A$. Round your answer to the nearest hundredth.

Solution:

$$\begin{aligned} DC &= AC - AD \\ DC &= \sqrt{89} - 5 \approx 4.43 \end{aligned}$$

Example 5: Determine if the triangle below is a right triangle. Explain why or why not.



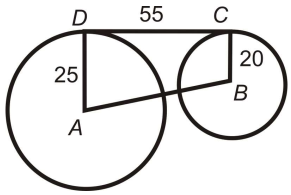
[Figure 12]

Solution: To determine if the triangle is a right triangle, use the Pythagorean Theorem. $4\sqrt{10}$ is the longest length, so we will set it equal to c in the formula.

$$\begin{aligned} 8^2 + 10^2 &\ ?\ (4\sqrt{10})^2 \\ 64 + 100 &\neq 160 \end{aligned}$$

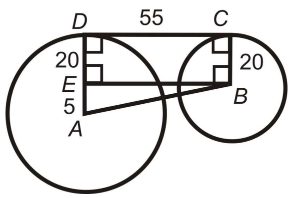
$\triangle ABC$ is not a right triangle. And, from the converse of the Tangent to a Circle Theorem, CB is not tangent to $\odot A$.

Example 6: Find the distance between the centers of the two circles. Reduce all radicals.



[Figure 13]

Solution: The distance between the two circles is AB . They are not tangent, however, $AD \perp DC$ and $DC \perp CB$. Let's add BE , such that $EDCB$ is a rectangle. Then, use the Pythagorean Theorem to find AB .

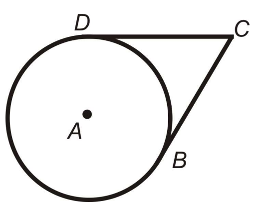


[Figure 14]

$$\begin{aligned} 5^2 + 55^2 &= AC^2 \\ 25 + 3025 &= AC^2 \\ 3050 &= AC^2 \\ AC &= \sqrt{3050} = 5\sqrt{122} \end{aligned}$$

Tangent Segments

Let's look at two tangent segments, drawn from the same external point. If we were to measure these two segments, we would find that they are equal.

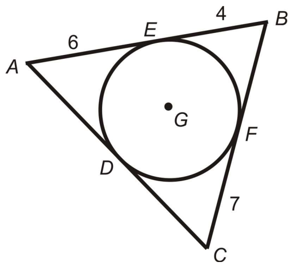


[Figure 15]

Theorem 10-2: If two tangent segments are drawn from the same external point, then the segments are equal.

The proof of Theorem 10-2 is in the review exercises.

Example 7: Find the perimeter of $\triangle ABC$.

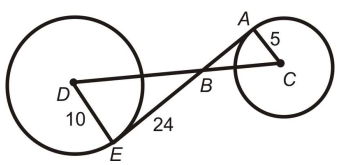


[Figure 16]

Solution: $AE = AD, EB = BF$, and $CF = CD$. Therefore, the perimeter of $\triangle ABC = 6+6+4+4+7+7 = 34$.

We say that $\odot G$ is ***inscribed*** in $\triangle ABC$. A circle is inscribed in a polygon, if every side of the polygon is tangent to the circle.

Example 8: If D and A are the centers and AE is tangent to both circles, find DC .



[Figure 17]

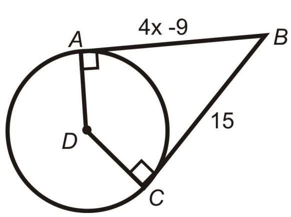
Solution: Because AE is tangent to both circles, it is perpendicular to both radii and $\triangle ABC$ and $\triangle DBE$ are similar. To find DB , use the Pythagorean Theorem.

$$\begin{aligned} 10^2 + 24^2 &= DB^2 \\ 100 + 576 &= 676 \\ DB &= \sqrt{676} = 26 \end{aligned}$$

To find BC , use similar triangles.

$$\frac{5}{10} = \frac{BC}{26} \longrightarrow BC = 13$$
$$DC = AB + BC = 26 + 13 = 39$$

Example 9: ***Algebra Connection*** Find the value of x .



[Figure 18]

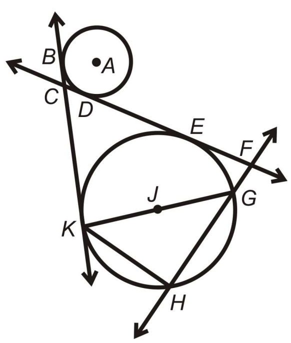
Solution: Because $AB \perp AD$ and $DC \perp CB, AB$ and CB are tangent to the circle and also congruent. Set $AB = CB$ and solve for x.

$$\begin{aligned} 4x - 9 &= 15 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

Know What? Revisited Refer to the photograph in the "Know What?" section at the beginning of this chapter. The orange line (which is normally black, but outlined for the purpose of this exercise) is a diameter of the smaller circle. Since this line passes through the center of the larger circle (yellow point, also outlined), it is part of one of its diameters. The "moon" hand is a diameter of the larger circle, but a secant of the smaller circle. The circles are not concentric because they do not have the same center and are not tangent because the sides of the circles do not touch.

Review Questions

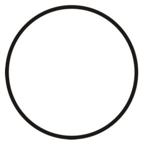
Determine which term best describes each of the following parts of $\odot P$.

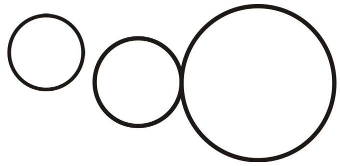


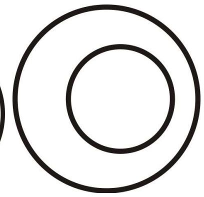
[Figure 19]

1. KG
2. $\overleftrightarrow{FH}$
3. KH
4. E
5. $\overleftrightarrow{BK}$
6. $\overleftrightarrow{CF}$
7. A
8. JG
9. What is the longest chord in any circle?

Copy each pair of circles. Draw in all common tangents.


[Figure 20]

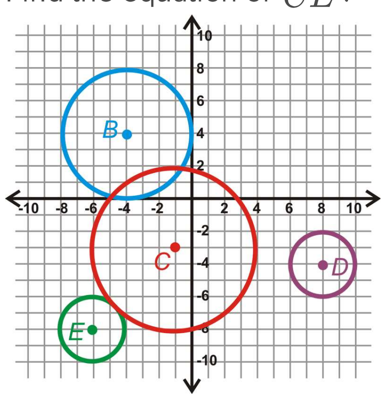

[Figure 21]


[Figure 22]

Coordinate Geometry Use the graph below to answer the following questions.

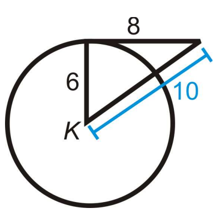
13. Find the radius of each circle.
14. Are any circles congruent? How do you know?
15. Find all the common tangents for $\odot B$ and $\odot C$.
16. $\odot C$ and $\odot E$ are externally tangent. What is CE?

Find the equation of $\overline{CE}$.

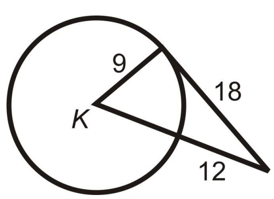


[Figure 23]

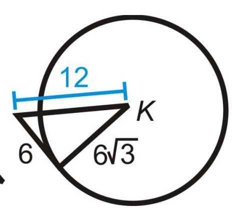
Determine whether the given segment is tangent to $\odot K$.



[Figure 24]

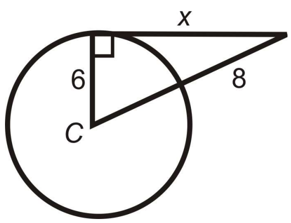


[Figure 25]

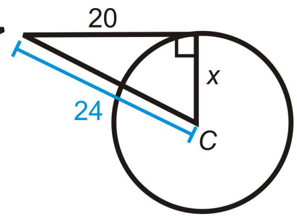


[Figure 26]

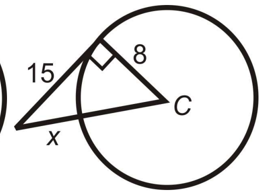
Algebra Connection Find the value of the indicated length(s) in $\odot C$. A and B are points of tangency. Simplify all radicals.



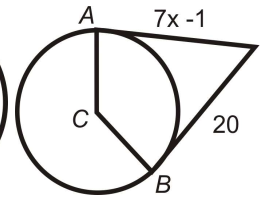
[Figure 27]



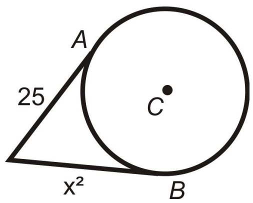
[Figure 28]



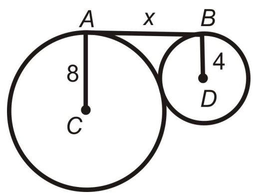
[Figure 29]



[Figure 30]

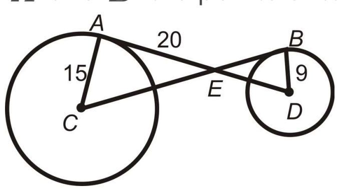


[Figure 31]



[Figure 32]

A and B are points of tangency for $\odot C$ and $\odot D$, respectively.



[Figure 33]

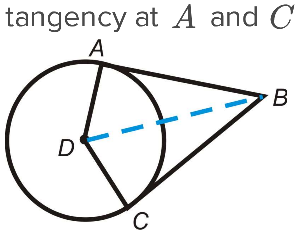
a. Is $\triangle AEC \sim \triangle BED$? Why?

b. Find BC.

c. Find AD.

d. Using the trigonometric ratios, find $m\angle C$. Round to the nearest tenth of a degree.

Fill in the blanks in the proof of Theorem 10-2. Given: AB and CB with points of tangency at A and C. AD and DC are radii. Prove: $AB \cong CB$



[Figure 34]

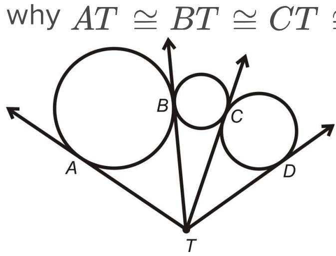
Statement	***Reason***
1.	
2. $AD \cong DC$	
3. $DA \perp AB$ and $DC \perp CB$	
4.	Definition of perpendicular lines
5.	Connecting two existing points
6. $\triangle ADB$ and $\triangle DCB$ are right triangles	
7. $DB \cong DB$	
8. $\triangle ABD \cong \triangle CBD$	
9. $AB \cong CB$	

29. From the above proof, we can also conclude (fill in the blanks):

a. $ABCD$ is a _______________ (type of quadrilateral).

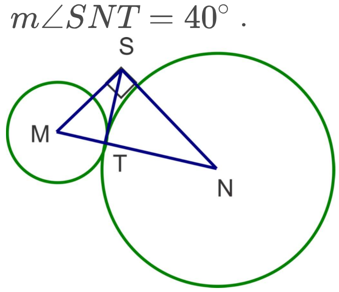
b. The line that connects the _____________ and the external point B ___________ $\angle ADC$ and $\angle ABC$.

Points A, B, C , and D are all points of tangency for the three tangent circles. *Explain* why $AT \cong BT \cong CT \cong DT$.



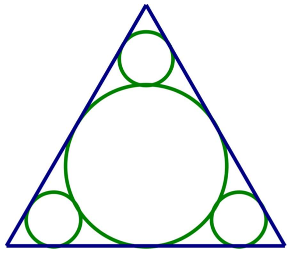
[Figure 35]

Circles tangent at T are centered at M and N . ST is tangent to both circles at T . Find the radius of the smaller circle if $SN \perp SM$, $SM = 22, TN = 25$ and $m\angle SNT = 40^\circ$.



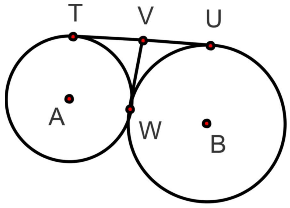
[Figure 36]

Four circles are arranged inside an equilateral triangle as shown. If the triangle has sides equal to 16 cm, what is the radius of the bigger circle? What are the radii of the smaller circles?



[Figure 37]

Circles centered at A and B are tangent at W. Explain why A, B and W are collinear. TU is a common external tangent to the two circles. VW is tangent to both circles. Justify the fact that $TV \cong VU \cong VW$.



[Figure 38]

Review Queue Answers

1. $y = -2x + 3$
2. $y = \frac{1}{3}x$
3. $y = -3x - 13$