3.2 Properties of Parallel Lines

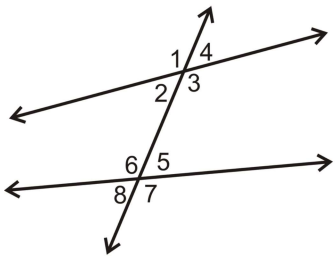
Last Modified: Dec 25, 2014

Learning Objectives

- Use the Corresponding Angles Postulate.
- Use the Alternate Interior Angles Theorem.
- Use the Alternate Exterior Angles Theorem.
- Use Same Side Interior Angles Theorem.

Review Queue

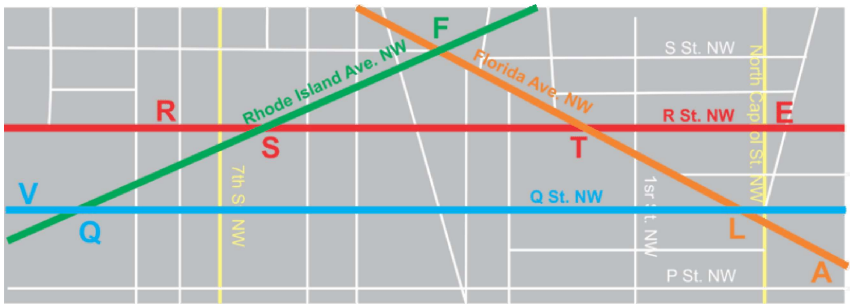
Use the picture below to determine:



[Figure 1]

1. A pair of corresponding angles.
2. A pair of alternate interior angles.
3. A pair of same side interior angles.
4. If $m\angle 4 = 37^\circ$, what other angles do you know?

Know What? The streets below are in Washington DC. The red street is R St. and the blue street is Q St. These two streets are parallel. The transversals are: Rhode Island Ave. (green) and Florida Ave. (orange).



[Figure 2]

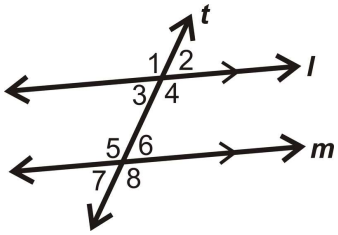
1. If $m\angle FTS = 35^\circ$, determine the other angles that are 35° .
2. If $m\angle SQV = 160^\circ$, determine the other angles that are 160° .
3. Why do you think the "State Streets" exists? Why aren't all the streets parallel or perpendicular?

In this section, we are going to discuss a specific case of two lines cut by a transversal. The two lines are now going to be parallel. If the two lines are parallel, all of the angles, corresponding, alternate interior, alternate exterior and same side interior have new properties. We will begin with corresponding angles.

Corresponding Angles Postulate

Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

If $l \,||\, m$ and both are cut by t, then $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.



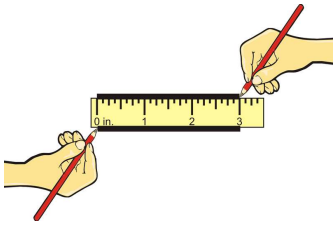
[Figure 3]

l **must be parallel** to m in order to use this postulate. Recall that a postulate is just like a theorem, but does not need to be proven. We can take it as true and use it just like a theorem from this point.

Investigation 3-4: Corresponding Angles Exploration

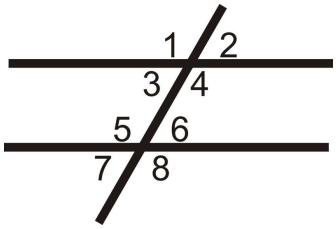
You will need: paper, ruler, protractor

Place your ruler on the paper. On either side of the ruler, draw lines, 3 inches long. This is the easiest way to ensure that the lines are parallel.



[Figure 4]

Remove the ruler and draw a transversal. Label the eight angles as shown.

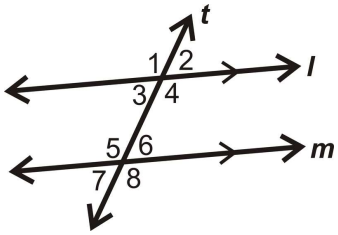


[Figure 5]

1. Using your protractor, measure all of the angles. What do you notice?

In this investigation, you should see that $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$ and $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$. $\angle 1 \cong \angle 4$, $\angle 5 \cong \angle 8$ by the Vertical Angles Theorem. By the Corresponding Angles Postulate, we can say $\angle 1 \cong \angle 5$ and therefore $\angle 1 \cong \angle 8$ by the Transitive Property. You can use this reasoning for the other set of congruent angles as well.

Example 1: If $m\angle 2 = 76^\circ$, what is $m\angle 6$?



[Figure 6]

Solution: $\angle 2$ and $\angle 6$ are corresponding angles and $l \parallel m$, from the markings in the picture. By the Corresponding Angles Postulate the two angles are equal, so $m\angle 6 = 76^\circ$.

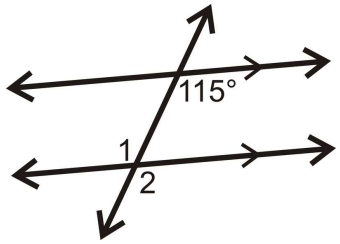
Example 2: Using the measures of $\angle 2$ and $\angle 6$ from Example 2, find all the other angle measures.

Solution: If $m\angle 2 = 76^\circ$, then $m\angle 1 = 180^\circ - 76^\circ = 104^\circ$ because they are a linear pair. $\angle 3$ is a vertical angle with $\angle 2$, so $m\angle 3 = 76^\circ$. $\angle 1$ and $\angle 4$ are vertical angles, so $m\angle 4 = 104^\circ$. By the Corresponding Angles Postulate, we know

$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$, so
$m\angle 5 = 104^\circ$, $m\angle 6 = 76^\circ$, $m\angle 7 = 76^\circ$, and $m\angle 104^\circ$.

Alternate Interior Angles Theorem

Example 3: Find $m\angle 1$.

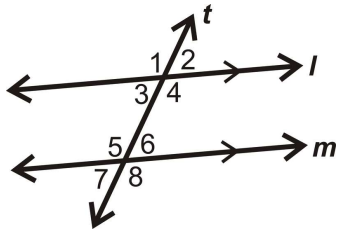


[Figure 7]

Solution: $m\angle 2 = 115^\circ$ because they are corresponding angles and the lines are parallel. $\angle 1$ and $\angle 2$ are vertical angles, so $m\angle 1 = 115^\circ$ also.

$\angle 1$ and the 115° angle are alternate interior angles.

Alternate Interior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.



[Figure 8]

Proof of Alternate Interior Angles Theorem

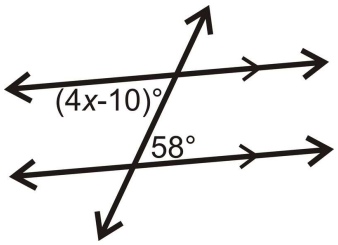
Given: $l \parallel m$

Prove: $\angle 3 \cong \angle 6$

Statement	*Reason*
1. $l \parallel m$	Given
2. $\angle 3 \cong \angle 7$	Corresponding Angles Postulate
3. $\angle 7 \cong \angle 6$	Vertical Angles Theorem
4. $\angle 3 \cong \angle 6$	Transitive PoC

There are several ways we could have done this proof. For example, Step 2 could have been $\angle 2 \cong \angle 6$ for the same reason, followed by $\angle 2 \cong \angle 3$. We could have also proved that $\angle 4 \cong \angle 5$.

Example 4: ***Algebra Connection*** Find the measure of the angle and x.



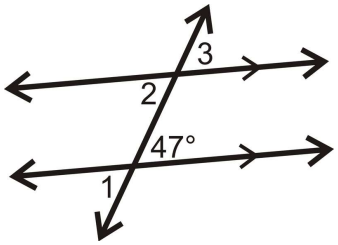
[Figure 9]

Solution: The two given angles are alternate interior angles so, they are equal. Set the two expressions equal to each other and solve for x.

$$\begin{aligned}(4x-10)^\circ &= 58^\circ \\ 4x &= 68^\circ \\ x &= 17^\circ\end{aligned}$$

Alternate Exterior Angles Theorem

Example 5: Find $m\angle 1$ and $m\angle 3$.



[Figure 10]

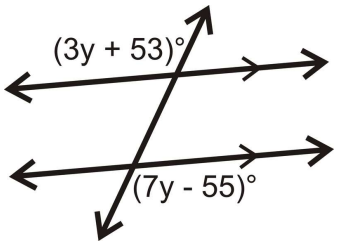
Solution: $m\angle 1 = 47^\circ$ because they are vertical angles. Because the lines are parallel, $m\angle 3 = 47^\circ$ by the Corresponding Angles Theorem. Therefore, $m\angle 2 = 47^\circ$.

$\angle 1$ and $\angle 3$ are alternate exterior angles.

Alternate Exterior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

The proof of this theorem is very similar to that of the Alternate Interior Angles Theorem and you will be asked to do in the exercises at the end of this section.

Example 6: ***Algebra Connection*** Find the measure of each angle and the value of y.



[Figure 11]

Solution: The given angles are alternate exterior angles. Because the lines are parallel, we can set the expressions equal to each other to solve the problem.

$$\begin{aligned}(3y+53)^\circ &= (7y-55)^\circ \\ 108^\circ &= 4y \\ 27^\circ &= y\end{aligned}$$

If $y = 27^\circ$, then each angle is $3(27^\circ) + 53^\circ$, or 134° .

Same Side Interior Angles Theorem

Same side interior angles have a different relationship that the previously discussed angle pairs.

Example 7: Find $m\angle 2$.

[Figure 12]

Solution: Here, $m\angle 1 = 66^\circ$ because they are alternate interior angles. $\angle 1$ and $\angle 2$ are a linear pair, so they are supplementary.

$$\begin{aligned}m\angle 1 + m\angle 2 &= 180^\circ \\ 66^\circ + m\angle 2 &= 180^\circ \\ m\angle 2 &= 114^\circ\end{aligned}$$

This example shows that if two parallel lines are cut by a transversal, the same side interior angles are supplementary.

Same Side Interior Angles Theorem: If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.

If $l \,||\, m$ and both are cut by t , then

$m\angle 3 + m\angle 5 = 180^\circ$ and $m\angle 4 + m\angle 6 = 180^\circ$.

[Figure 13]

You will be asked to do the proof of this theorem in the review questions.

Example 8: ***Algebra Connection*** Find the measure of x.

[Figure 14]

Solution: The given angles are same side interior angles. The lines are parallel, therefore the angles add up to 180°. Write an equation.

$$\begin{aligned}(2x+43)^\circ+(2x-3)^\circ &= 180^\circ\\ (4x+40)^\circ &= 180^\circ\\ 4x &= 140^\circ\\ x &= 35^\circ\end{aligned}$$

While you might notice other angle relationships, there are no more theorems to worry about. However, we will continue to explore these other angle relationships. For example, same side exterior angles are also supplementary. You will prove this in the review questions.

Example 9: $l \,||\, m$ and $s \,||\, t$. Prove $\angle 1 \cong \angle 16$.

[Figure 15]

Solution:

Statement	*Reason*
1. $l \,||\, m$ and $s \,||\, t$	Given
2. $\angle 1 \cong \angle 3$	Corresponding Angles Postulate
3. $\angle 3 \cong \angle 16$	Alternate Exterior Angles Theorem
4. $\angle 1 \cong \angle 16$	Transitive PoC

Know What? Revisited Using what we have learned in this lesson, the other angles that are 35° are $\angle TLQ$, $\angle ETL$, and the vertical angle with $\angle TLQ$. The other angles that are 160° are $\angle FSR$, $\angle TSQ$, and the vertical angle with $\angle SQV$. You could argue that the "State Streets" exist to help traffic move faster and more efficiently through the city.

Review Questions

For questions 1-7, determine if each angle pair below is congruent, supplementary or neither.

[Figure 16]

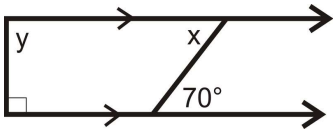
1. $\angle 1$ and $\angle 7$
2. $\angle 4$ and $\angle 2$
3. $\angle 6$ and $\angle 3$
4. $\angle 5$ and $\angle 8$
5. $\angle 1$ and $\angle 6$
6. $\angle 4$ and $\angle 6$
7. $\angle 2$ and $\angle 3$

For questions 8-16, determine if the angle pairs below are: Corresponding Angles, Alternate Interior Angles, Alternate Exterior Angles, Same Side Interior Angles, Vertical Angles, Linear Pair or None.

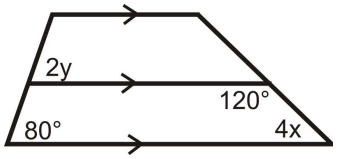
[Figure 17]

8. $\angle 2$ and $\angle 13$
9. $\angle 7$ and $\angle 12$
10. $\angle 1$ and $\angle 11$
11. $\angle 6$ and $\angle 10$
12. $\angle 14$ and $\angle 9$
13. $\angle 3$ and $\angle 11$
14. $\angle 4$ and $\angle 15$
15. $\angle 5$ and $\angle 16$
16. List all angles congruent to $\angle 8$.

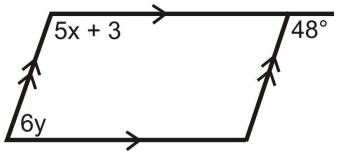
For 17-20, find the values of x and y.



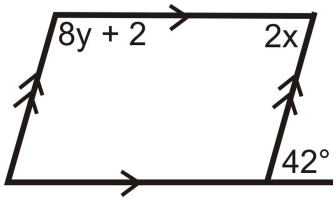
[Figure 18]



[Figure 19]

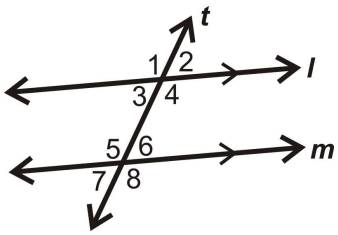


[Figure 20]



[Figure 21]

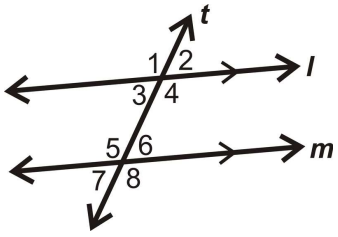
Algebra Connection For questions 21-25, use the picture below. Find the value of x and/or y.



[Figure 22]

21. $m\angle 1 = (4x + 35)^\circ$, $m\angle 8 = (7x - 40)^\circ$
22. $m\angle 2 = (3y + 14)^\circ$, $m\angle 6 = (8x - 76)^\circ$
23. $m\angle 3 = (3x + 12)^\circ$, $m\angle 5 = (5x + 8)^\circ$
24. $m\angle 4 = (5x - 33)^\circ$, $m\angle 5 = (2x + 60)^\circ$
25. $m\angle 1 = (11y - 15)^\circ$, $m\angle 7 = (5y + 3)^\circ$

Fill in the blanks in the proof below.

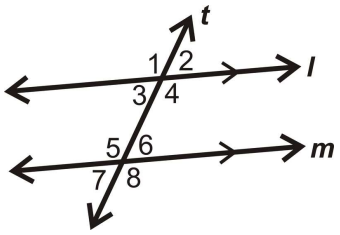


[Figure 23]

Given: $l \parallel m$ Prove: $\angle 3$ and $\angle 5$ are supplementary (Same Side Interior Angles Theorem)

Statement	*Reason*
1.	Given
2. $\angle 1 \cong \angle 5$	
3.	$\cong$ angles have = measures
4.	Linear Pair Postulate
5.	Definition of Supplementary Angles
6. $m\angle 3 + m\angle 5 = 180^\circ$	
7. $\angle 3$ and $\angle 5$ are supplementary	

For 27 and 28, use the picture to the right to complete each proof.

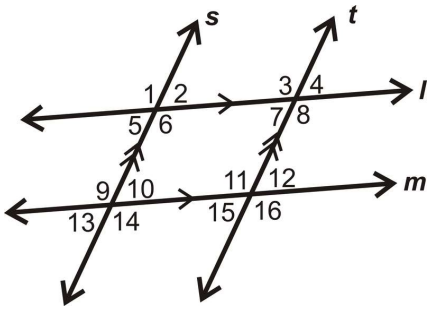


[Figure 24]

27. Given: $l \,||\, m$ Prove: $\angle 1 \cong \angle 8$ (Alternate Exterior Angles Theorem)

28. Given: $l \,||\, m$ Prove: $\angle 2$ and $\angle 8$ are supplementary

For 29-31, use the picture to the right to complete each proof.



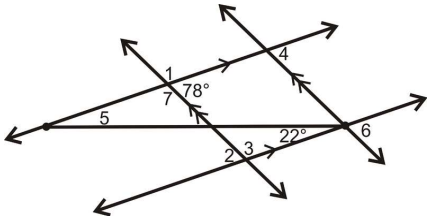
[Figure 25]

29. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 4 \cong \angle 10$

30. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 2 \cong \angle 15$

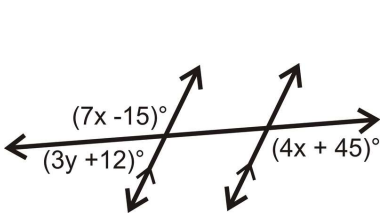
31. Given: $l \,||\, m,\ s \,||\, t$ Prove: $\angle 4$ and $\angle 9$ are supplementary

Find the measures of all the numbered angles in the figure below.

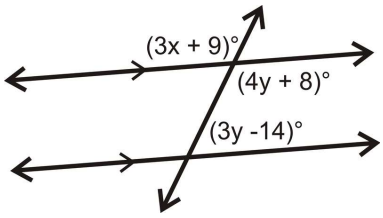


[Figure 26]

Algebra Connection For 33 and 34, find the values of x and y.



[Figure 27]



[Figure 28]

33. ***Error Analysis*** Nadia is working on Problem 31. Here is her proof:

Statement	*Reason*
1. $l \parallel m,\ s \parallel t$	Given
2. $\angle 4 \cong \angle 15$	Alternate Exterior Angles Theorem
3. $\angle 15 \cong \angle 14$	Same Side Interior Angles Theorem
4. $\angle 14 \cong \angle 9$	Vertical Angles Theorem
5. $\angle 4 \cong \angle 9$	Transitive PoC

What happened? Explain what is needed to be done to make the proof correct.

Review Queue Answers

1. $\angle 1$ and $\angle 6, \angle 2$ and $\angle 8, \angle 3$ and $\angle 7$, or $\angle 4$ and $\angle 5$
2. $\angle 2$ and $\angle 5$ or $\angle 3$ and $\angle 6$
3. $\angle 1$ and $\angle 7$ or $\angle 4$ and $\angle 8$
4. $\angle 3$ and $\angle 5$ or $\angle 2$ and $\angle 6$