

2-5

Reasoning in Algebra and Geometry

Objective To connect reasoning in algebra and geometry



Think about how each step is related to the steps before it.

Getting Ready!

Follow the steps of the brainteaser using your age. Then try it using a family member's age. What do you notice? Explain how the brainteaser works.

- Write down your age.
- Multiply it by 10.
- Add 8 to the product.
- Double that answer and then subtract 16.
- Finally, divide the result by 2.



Lesson Vocabulary

- Reflexive Property
- Symmetric Property
- Transitive Property
- proof
- two-column proof

In the Solve It, you logically examined a series of steps. In this lesson, you will apply logical reasoning to algebraic and geometric situations.

Essential Understanding Algebraic properties of equality are used in geometry. They will help you solve problems and justify each step you take.

In geometry you accept postulates and properties as true. Some of the properties that you accept as true are the properties of equality from algebra.



Key Concept Properties of Equality

Let a , b , and c be any real numbers.

Addition Property

If $a = b$, then $a + c = b + c$.

Subtraction Property

If $a = b$, then $a - c = b - c$.

Multiplication Property

If $a = b$, then $a \cdot c = b \cdot c$.

Division Property

If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Reflexive Property

$a = a$

Symmetric Property

If $a = b$, then $b = a$.

Transitive Property

If $a = b$ and $b = c$, then $a = c$.

Substitution Property

If $a = b$, then b can replace a in any expression.

take note

Key Concept The Distributive Property

Use multiplication to distribute a to each term of the sum or difference within the parentheses.

Sum:

$$a(b + c) = a(b + c) = ab + ac$$

Difference:

$$a(b - c) = a(b - c) = ab - ac$$

You use deductive reasoning when you solve an equation. You can justify each step with a postulate, a property, or a definition. For example, you can use the Distributive Property to justify combining like terms. If you think of the Distributive Property as $ab + ac = a(b + c)$ or $ab + ac = (b + c)a$, then $2x + x = (2 + 1)x = 3x$.

Plan

How can you use the given information?

Use what you know about linear pairs to relate the two angles.



Problem 1 Justifying Steps When Solving an Equation

Algebra What is the value of x ? Justify each step.

$\angle AOM$ and $\angle MOC$ are supplementary.

\angle that form a linear pair are supplementary.

$$m\angle AOM + m\angle MOC = 180$$

Definition of supplementary \angle

$$(2x + 30) + x = 180$$

Substitution Property

$$3x + 30 = 180$$

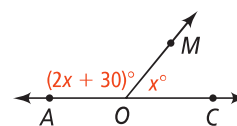
Distributive Property

$$3x = 150$$

Subtraction Property of Equality

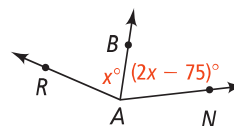
$$x = 50$$

Division Property of Equality



Got It? 1. What is the value of x ? Justify each step.

Given: \overrightarrow{AB} bisects $\angle RAN$.



Some properties of equality have corresponding properties of congruence.

take note

Key Concept Properties of Congruence

Reflexive Property

$$\overline{AB} \cong \overline{AB} \quad \angle A \cong \angle A$$

Symmetric Property

$$\text{If } \overline{AB} \cong \overline{CD}, \text{ then } \overline{CD} \cong \overline{AB}.$$

$$\text{If } \angle A \cong \angle B, \text{ then } \angle B \cong \angle A.$$

Transitive Property

$$\text{If } \overline{AB} \cong \overline{CD} \text{ and } \overline{CD} \cong \overline{EF}, \text{ then } \overline{AB} \cong \overline{EF}.$$

$$\text{If } \angle A \cong \angle B \text{ and } \angle B \cong \angle C, \text{ then } \angle A \cong \angle C.$$

$$\text{If } \angle B \cong \angle A \text{ and } \angle B \cong \angle C, \text{ then } \angle A \cong \angle C.$$

Think

Is the justification a property of equality or congruence?

Numbers are equal ($=$) and you can perform operations on them, so (A) and (C) are properties of equality. Figures and their corresponding parts are congruent (\cong), so (B) is a property of congruence.



Problem 2 Using Properties of Equality and Congruence

What is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

- A** $2x + 9 = 19$
 $2x = 10$ Subtraction Property of Equality
- B** $\angle O \cong \angle W$ and $\angle W \cong \angle L$
 $\angle O \cong \angle L$ Transitive Property of Congruence
- C** $m\angle E = m\angle T$
 $m\angle T = m\angle E$ Symmetric Property of Equality



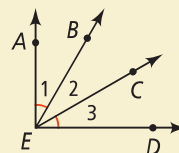
Got It? 2. For parts (a)–(c), what is the name of the property of equality or congruence that justifies going from the first statement to the second statement?

- a. $\overline{AR} \cong \overline{TY}$
 $\overline{TY} \cong \overline{AR}$
- b. $3(x + 5) = 9$
 $3x + 15 = 9$
- c. $\frac{1}{4}x = 7$
 $x = 28$
- d. **Reasoning** What property justifies the statement $m\angle R = m\angle R$?

A **proof** is a convincing argument that uses deductive reasoning. A proof logically shows why a conjecture is true. A **two-column proof** lists each statement on the left. The justification, or the reason for each statement, is on the right. Each statement must follow logically from the steps before it. The diagram below shows the setup for a two-column proof. You will find the complete proof in Problem 3.

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle AEC = m\angle DEB$



The first statement is usually the given statement.

Each statement should follow logically from the previous statements.

The last statement is what you want to prove.

Statements	Reasons
1) $m\angle 1 = m\angle 3$	1) Given
2) ~~~~~	2) ~~~~~
3) ~~~~~	3) ~~~~~
4) ~~~~~	4) ~~~~~
5) $m\angle AEC = m\angle DEB$	5) ~~~~~

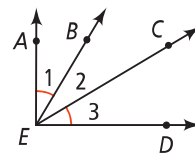


Problem 3 Writing a Two-Column Proof

Write a two-column proof.

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle AEC = m\angle DEB$



Know

$m\angle 1 = m\angle 3$

Need

To prove that
 $m\angle AEC = m\angle DEB$

Plan

Add $m\angle 2$ to both $m\angle 1$ and $m\angle 3$. The resulting angles will have equal measure.

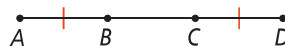
Statements	Reasons
1) $m\angle 1 = m\angle 3$	1) Given
2) $m\angle 2 = m\angle 2$	2) Reflexive Property of Equality
3) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3) Addition Property of Equality
4) $m\angle 1 + m\angle 2 = m\angle AEC$ $m\angle 3 + m\angle 2 = m\angle DEB$	4) Angle Addition Postulate
5) $m\angle AEC = m\angle DEB$	5) Substitution Property



Got It? 3. a. Write a two-column proof.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



b. **Reasoning** In Problem 3, why is Statement 2 necessary in the proof?



Lesson Check

Do you know HOW?

Name the property of equality or congruence that justifies going from the first statement to the second statement.

- $m\angle A = m\angle S$ and $m\angle S = m\angle K$
 $m\angle A = m\angle K$
- $3x + x + 7 = 23$
 $4x + 7 = 23$
- $4x + 5 = 17$
 $4x = 12$

Do you UNDERSTAND?

4. **Developing Proof** Fill in the reasons for this algebraic proof.

Given: $5x + 1 = 21$

Prove: $x = 4$

Statements	Reasons
1) $5x + 1 = 21$	1) a. ?
2) $5x = 20$	2) b. ?
3) $x = 4$	3) c. ?



Practice and Problem-Solving Exercises



Practice **Algebra** Fill in the reason that justifies each step.

See Problem 1.

5. $\frac{1}{2}x - 5 = 10$ Given
 $2(\frac{1}{2}x - 5) = 20$ a. ?
 $x - 10 = 20$ b. ?
 $x = 30$ c. ?

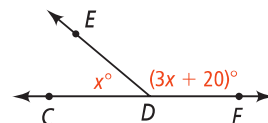
6. $5(x + 3) = -4$ Given
 $5x + 15 = -4$ a. ?
 $5x = -19$ b. ?
 $x = -\frac{19}{5}$ c. ?

7. $\angle CDE$ and $\angle EDF$ are supplementary.

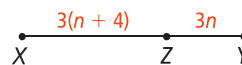
$$\begin{aligned} m\angle CDE + m\angle EDF &= 180 \\ x + (3x + 20) &= 180 \\ 4x + 20 &= 180 \\ 4x &= 160 \\ x &= 40 \end{aligned}$$

\angle s that form a linear pair are supplementary

- a. ?
b. ?
c. ?
d. ?
e. ?



8. $XY = 42$ Given
 $XZ + ZY = XY$ a. ?
 $3(n + 4) + 3n = 42$ b. ?
 $3n + 12 + 3n = 42$ c. ?
 $6n + 12 = 42$ d. ?
 $6n = 30$ e. ?
 $n = 5$ f. ?



Name the property of equality or congruence that justifies going from the first statement to the second statement.

See Problem 2.

9. $2x + 1 = 7$
 $2x = 6$

10. $5x = 20$
 $x = 4$

11. $\overline{ST} \cong \overline{QR}$
 $\overline{QR} \cong \overline{ST}$

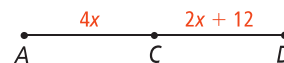
12. $AB - BC = 12$
 $AB = 12 + BC$

13. **Developing Proof** Fill in the missing statements or reasons for the following two-column proof.

See Problem 3.

Given: C is the midpoint of \overline{AD} .

Prove: $x = 6$



Statements	Reasons
1) C is the midpoint of \overline{AD} .	1) a. <u>?</u>
2) $\overline{AC} \cong \overline{CD}$	2) b. <u>?</u>
3) $AC = CD$	3) \cong segments have equal length.
4) $4x = 2x + 12$	4) c. <u>?</u>
5) d. <u>?</u>	5) Subtraction Property of Equality
6) $x = 6$	6) e. <u>?</u>

B Apply

Use the given property to complete each statement.

14. Symmetric Property of Equality

If $AB = YU$, then $\underline{\quad?}$.

16. Reflexive Property of Congruence

$\angle POR \cong \underline{\quad?}$

18. Substitution Property

If $LM = 7$ and $EF + LM = NP$,
then $\underline{\quad?}$ = NP .

15. Symmetric Property of Congruence

If $\angle H \cong \angle K$, then $\underline{\quad?}$ $\cong \angle H$.

17. Distributive Property

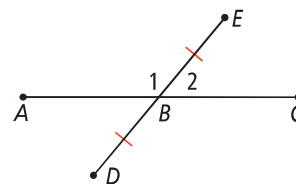
$3(x - 1) = 3x - \underline{\quad?}$

19. Transitive Property of Congruence

If $\angle XYZ \cong \angle AOB$ and
 $\angle AOB \cong \angle WYT$, then $\underline{\quad?}$.

20. **Think About a Plan** A very important part in writing proofs is analyzing the diagram for key information. What true statements can you make based on the diagram at the right?

- What theorems or definitions relate to the geometric figures in the diagram?
- What types of markings show relationships between parts of geometric figures?



21. **Writing** Explain why the statements $\overline{LR} \cong \overline{RL}$ and $\angle CBA \cong \angle ABC$ are both true by the Reflexive Property of Congruence.

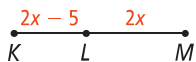
22. **Reasoning** Complete the following statement. Describe the reasoning that supports your answer.

The Transitive Property of Falling Dominoes: If Domino A causes Domino B to fall, and Domino B causes Domino C to fall, then Domino A causes Domino $\underline{\quad?}$ to fall.

Write a two-column proof.

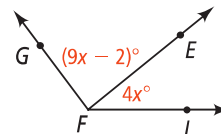
23. **Given:** $KM = 35$

Proof Prove: $KL = 15$



24. **Given:** $m\angle GFI = 128$

Proof Prove: $m\angle EFI = 40$



C Challenge

25. **Error Analysis** The steps below “show” that $1 = 2$. Describe the error.

$a = b$	Given
$ab = b^2$	Multiplication Property of Equality
$ab - a^2 = b^2 - a^2$	Subtraction Property of Equality
$a(b - a) = (b + a)(b - a)$	Distributive Property
$a = b + a$	Division Property of Equality
$a = a + a$	Substitution Property
$a = 2a$	Simplify.
$1 = 2$	Division Property of Equality

Relationships Consider the following relationships among people. Tell whether each relationship is *reflexive*, *symmetric*, *transitive*, or *none of these*. Explain.

Sample: The relationship “is younger than” is not reflexive because Sue is not younger than herself. It is not symmetric because if Sue is younger than Fred, then Fred is not younger than Sue. It is transitive because if Sue is younger than Fred and Fred is younger than Alana, then Sue is younger than Alana.

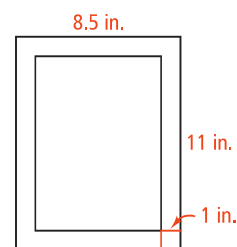
26. has the same birthday as 27. is taller than 28. lives in a different state than

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

29. You are typing a one-page essay for your English class. You set 1-in. margins on all sides of the page as shown in the figure at the right. How many square inches of the page will contain your essay?
30. Given $2(m\angle A) + 17 = 45$ and $m\angle B = 2(m\angle A)$, what is $m\angle B$?
31. A circular flowerbed has circumference 14π m. What is its area in square meters? Use 3.14 for π .
32. The measure of the supplement of $\angle 1$ is 98. What is $m\angle 1$?
33. What is the next term in the sequence 2, 4, 8, 14, 22, 32, 44, ... ?



Mixed Review

34. **Reasoning** Use logical reasoning to draw a conclusion.

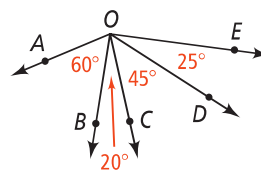
If a student is having difficulty in class, then that student's teacher is concerned.

Walt is having difficulty in science class.

See Lesson 2-4.

Use the diagram at the right. Find each measure.

35. $m\angle AOC$ 36. $m\angle DOB$
37. $m\angle AOD$ 38. $m\angle BOE$



See Lesson 1-4.

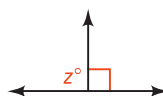
Get Ready! To prepare for Lesson 2-6, do Exercises 39–41.

Find the value of each variable.

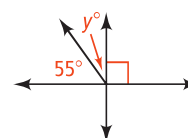
39.



40.



41.



See Lesson 1-5.