1.2 Segments and Distance

Difficulty Level: **At Grade** | Created by: CK-12
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Learning Objectives

- Understand the ruler postulate.
- Understand the segment addition postulate.
- Place line segments on a coordinate grid.

Review Queue

Answer the following questions.

How would you label the following geometric figure? List 3 different ways

[Figure 1]

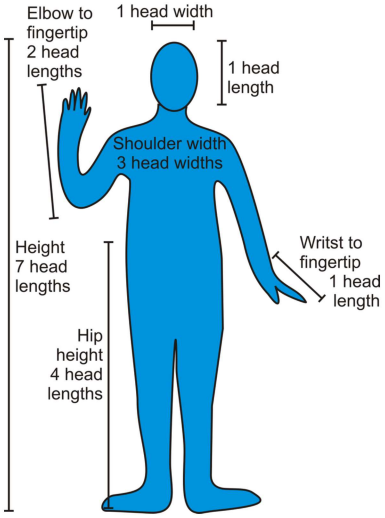
1. Draw three collinear points and a fourth that is coplanar with these points.
2. Plot the following points on the $x - y$ plane.
   a. (3, -3)
   b. (-4, 2)
   c. (0, -7)
   d. (6, 0)
3. Find the equation of the line containing the points (-4, 3) and (6, -2).

Know What? The average adult human body can be measured in “heads.” For example, the average human is 7-8 heads tall. When doing this, keep in mind that each person uses their own head to measure their own body. Other interesting measurements are in the picture to the right.

After analyzing the picture, we can determine a few other measurements that aren’t listed.

- The length from the wrist to the elbow
- The length from the top of the neck to the hip
- The width of each shoulder

What are these measurements?



[Figure 2]

Measuring Distances

Distance: The length between two points.

Measure: To determine how far apart two geometric objects are.

The most common way to measure distance is with a ruler. In this class we will use both inches and centimeters.

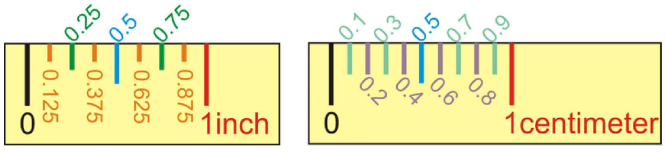
Example 1: Determine how long the line segment is, in inches. Round to the nearest quarter-inch.

[Figure 3]

Solution: To measure this line segment with a ruler, it is very important to line up the "0" with the one of the endpoints. DO NOT USE THE EDGE OF THE RULER. This segment is about 3.5 inches (in) long.

As a reminder, inch-rulers are usually divided up by $\frac{1}{8}$-in. (or 0.125 in) segments.

Centimeter rulers are divided up by $\frac{1}{10}$-centimenter (or 0.1 cm) segments.



[Figure 4]

The two rulers above are NOT DRAWN TO SCALE. Anytime you see this statement, it means that the measured length is not actually the distance apart that it is labeled. Different problems and examples will be labeled this way because it can be difficult to draw problems in this text to full scale. You should never assume that objects are drawn to scale. Always rely on the measurements or markings given in a diagram.

Example 2: Determine the measurement between the two points to the nearest tenth of a centimeter.

[Figure 5]

Solution: Even though there is no line segment between the two points, we can still measure the distance using a ruler. It looks like the two points are 4.5 centimeters (cm) apart.

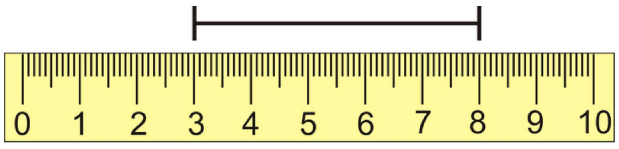
NOTE: We label a line segment, $\overline{AB}$. The ***distance*** between A and B is labeled as AB or $m\overline{AB}$, where m means measure. AB and $m\overline{AB}$ can be used interchangeably. In this text we will primarily use the first.

Ruler Postulate

Ruler Postulate: The distance between two points will be the absolute value of the difference between the numbers shown on the ruler.

The ruler postulate implies that you do not need to start measuring at "0", as long as you subtract the first number from the second. "Absolute value" is used because ***distance is always positive.***

Example 3: What is the distance marked on the ruler below? The ruler is in centimeters.



[Figure 6]

Solution: Find the absolute value of difference between the numbers shown. The line segment spans from 3 cm to 8 cm.

$$|8-3| = |5| = 5$$

The line segment is 5 cm long. Notice that you also could have done $|3 - 8| = |-5| = 5$.

Example 4: Draw $\overline{CD}$, such that $CD = 3.825\ in$.

Solution: To draw a line segment, start at "0" and draw a segment to 3.825 in. Put points at each end and label.

[Figure 7]

Segment Addition Postulate

Before we introduce this postulate, we need to address what the word "between" means in geometry.

[Figure 8]

B is between A and C in this picture. As long as B is ***anywhere on the segment***, it can be considered to be ***between*** the endpoints.

Segment Addition Postulate: If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

The picture above illustrates the Segment Addition Postulate. If $AB = 5\ cm$ and $BC = 12\ cm$, then AC must equal $5 + 12$ or 17 cm. You may also think of this as the "sum of the partial lengths, will be equal to the whole length."

Example 5: Make a sketch of $\overline{OP}$, where Q is between O and P.

Solution: Draw $\overline{OP}$ first, then place Q somewhere along the segment.

[Figure 9]

Example 6: In the picture from Example 5, if $OP = 17$ and $QP = 6$, what is OQ?

Solution: Use the Segment Additional Postulate. $OQ + QP = OP$, so $OQ + 6 = 17$, or $OQ = 17 - 6 = 11$. So, $OQ = 11$.

Example 7: Make a sketch that matches the description: S is between T and V. R is between S and T. $TR = 6\ cm$, $RV = 23\ cm$, and $TR = SV$. Then, find SV, TS, RS and TV.

Solution: Interpret the first sentence first: S is between T and V.

[Figure 10]

Then add in what we know about R: It is between S and T.

[Figure 11]

To find SV, we know it is equal to TR, so $SV = 6\ cm$.

$$\begin{aligned} \underline{For\ RS}: RV &= RS + SV \\ 23 &= RS + 6 \\ RS &= 17\ cm \end{aligned} \qquad \begin{aligned} \underline{For\ TS}: TS &= TR + RS \\ TS &= 6 + 17 \\ TS &= 23\ cm \end{aligned} \qquad \begin{aligned} \underline{For\ TV}: TV &= TR + \\ TV &= 6 + 17 \\ TV &= 29\ cm \end{aligned}$$

Example 8: ***Algebra Connection*** For $\overline{HK}$, suppose that J is between H and K. If $HJ = 2x + 4,\ JK = 3x + 3,$ and $KH = 22$, find the lengths of HJ and JK.

Solution: Use the Segment Addition Postulate and then substitute what we know.

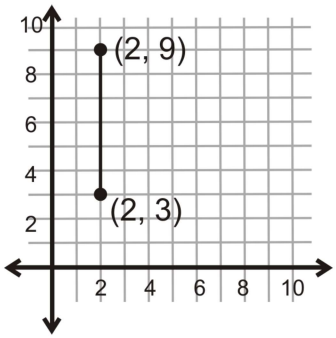
$$\begin{aligned} HJ + JK &= KH \\ (2x + 4) + (3x + 3) &= 22 \\ 5x + 7 &= 22 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

So, if $x = 3$, then $HJ = 10$ and $JK = 12$.

Distances on a Grid

In Algebra, you worked with graphing lines and plotting points in the $x - y$ plane. At this point, you can find the distances between points plotted in the $x - y$ plane if the lines are horizontal or vertical. ***If the line is vertical, find the change in the*** $y-$ ***coordinates. If the line is horizontal, find the change in the*** $x-$ ***coordinates.***

Example 8: What is the distance between the two points shown below?



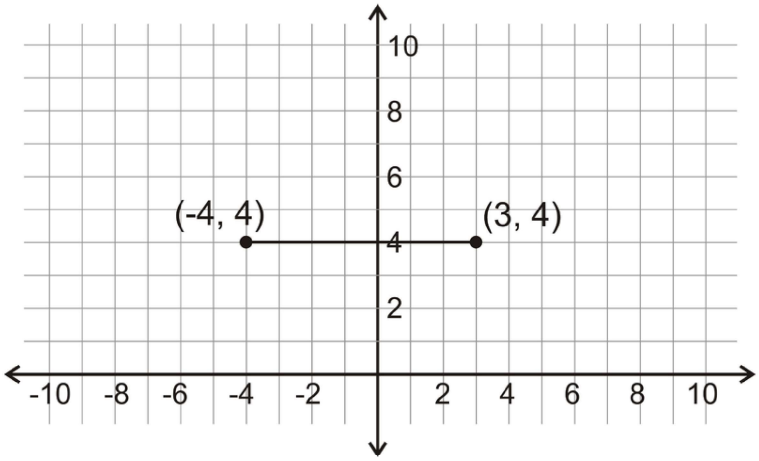
[Figure 12]

Solution: Because this line is vertical, look at the change in the $y-$ coordinates.

$$|9-3| = |6| = 6$$

The distance between the two points is 6 units.

Example 9: What is the distance between the two points shown below?



[Figure 13]

Solution: Because this line is horizontal, look at the change in the $x-$ coordinates.

$$|(-4)-3| = |-7| = 7$$

The distance between the two points is 7 units.

Know What? Revisited The length from the wrist to the elbow is one head, the length from the top of the neck to the hip is two heads, and the width of each shoulder one head width. There are several other interesting body proportion measurements. For example, your foot

is the same length as your forearm (wrist to elbow, on the interior of the arm).There are also facial proportions. All of these proportions are what artists use to draw the human body and what da Vinci used to draw his Vitruvian Man, http://en.wikipedia.org/wiki/Vitruvian_Man.

Review Questions

Find the length of each line segment in inches. Round to the nearest $\frac{1}{8}$ of an inch.

[Figure 14]

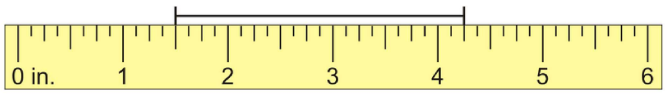
[Figure 15]

Find the distance between each pair of points in centimeters. Round to the nearest tenth.

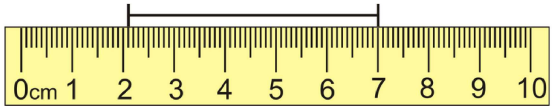
[Figure 16]

[Figure 17]

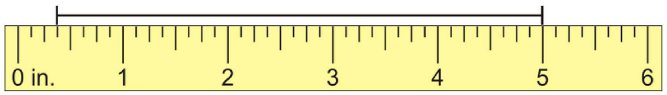
For 5-8, use the ruler in each picture to determine the length of the line segment.



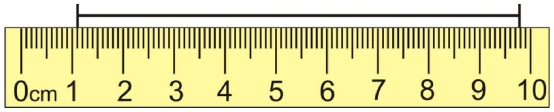
[Figure 18]



[Figure 19]



[Figure 20]



[Figure 21]

5. Make a sketch of $\overline{BT}$, with A between B and T.
6. If O is in the middle of $\overline{LT}$, where exactly is it located? If $LT = 16\ cm$, what is LO and OT?
7. For three collinear points, A between T and Q.
   a. Draw a sketch.
   b. Write the Segment Addition Postulate.
   c. If $AT = 10\ in$ and $AQ = 5\ in$, what is TQ?
8. For three collinear points, M between H and A.
   a. Draw a sketch.
   b. Write the Segment Addition Postulate.
   c. If $HM = 18\ cm$ and $HA = 29\ cm$, what is AM?
9. Make a sketch that matches the description: B is between A and D. C is between B and D. $AB = 7\ cm$, $AC = 15\ cm$, and $AD = 32\ cm$. Find BC, BD, and CD.
10. Make a sketch that matches the description: E is between F and G. H is between F and E. $FH = 4\ in$, $EG = 9\ in$, and $FH = HE$. Find FE, HG, and FG.
11. Make a sketch that matches the description: S is between T and V. R is between S and T. T is between R and Q. $QV = 18$, $QT = 6$, and $TR = RS = SV$.
    a. Find RS.
    b. Find QS.
    c. Find TS.
    d. Find TV.

For 16-20, Suppose J is between H and K. Use the Segment Addition Postulate to solve for x. Then find the length of each segment.

16. $HJ = 4x + 9,\ JK = 3x + 3,\ KH = 33$
17. $HJ = 5x - 3,\ JK = 8x - 9,\ KH = 131$

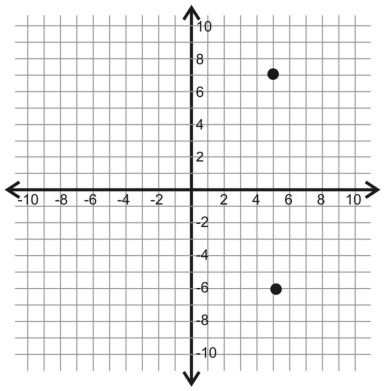
18. $HJ = 2x + \frac{1}{3},\ JK = 5x + \frac{2}{3},\ KH = 12x - 4$

19. $HJ = x + 10,\ JK = 9x,\ KH = 14x - 58$

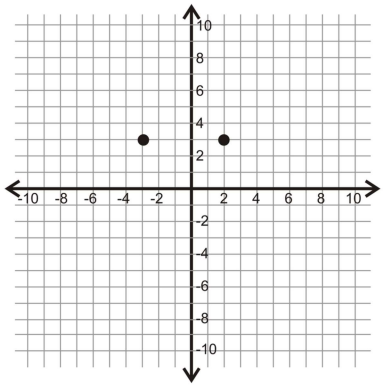
20. $HJ = \frac{3}{4}x - 5,\ JK = x - 1,\ KH = 22$

21. Draw four points, A, B, C, and D such that $AB = BC = AC = AD = BD$ (HINT: A, B, C and D should NOT be collinear)

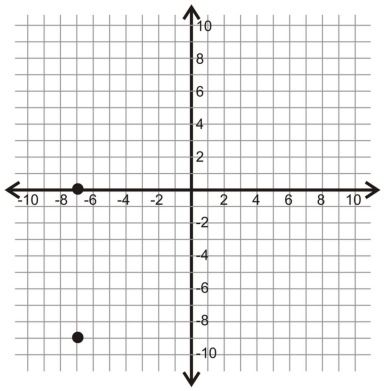
For 22-25, determine the vertical or horizontal distance between the two points.



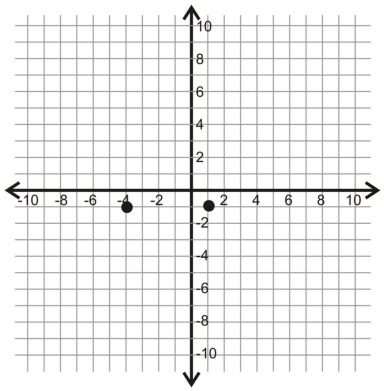
[Figure 22]



[Figure 23]



[Figure 24]



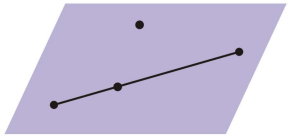
[Figure 25]

Each of the following problems presents an opportunity for students to extend their knowledge of measurement to the real world. Each of these concepts could be further developed into a mini-project.

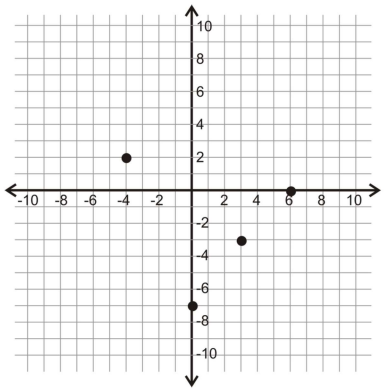
26. Measure the length of your head and create a "ruler" of this length out of cardstock or cardboard. Use your ruler to measure your height. Share your height in terms of your head length with your class and compare results.
27. Describe the advantages of using the metric system to measure length over the English system. Use the examples of the two rulers (one in inches and one in centimeters) to aid in your description.
28. A speedometer in a car measures distance traveled by tracking the number of rotations on the wheels on the car. A pedometer is a device that a person can wear that tracks the number of steps a person takes and calculates the distance traveled based on the person's stride length. Which would produce a more accurate measure of distance? Why? What could you do to make the less accurate measure more precise?
29. Research the origins of ancient measurement units such as the cubit. Research the origins of the units of measure we use today such as: foot, inch, mile, meter. Why are standard units important?
30. Research the facial proportions that da Vinci used to create his Vitruvian man. Write a summary of your findings.

Review Queue Answers

1. line $l, \overline{MN}, \overline{NM}$



[Figure 26]



[Figure 27]

$$m = \frac{3-(-2)}{-4-6} = \frac{5}{-10} = -\frac{1}{2}$$

2.

$$\begin{aligned} y &= -\frac{1}{2}x + b \\ -2 &= -\frac{1}{2}(6) + b \\ -2 &= -3 + b \\ 1 &= b \end{aligned}$$

So, the equation is $y = -\frac{1}{2}x + 1$